Springs

We need 2 formulas for Springs:

Hooke's Law

$F = kx$ & $W = \int_a^b kx\,dx$

distance from natural length

where k = spring constant

Examples Inspired by Thomas' Calculus 10th Edition

$\boxed{\text{Ex1}}$ If it takes 1800 J of work to stretch a spring from its natural length of 200 cm to a length of 500 cm, find the force required to stretch it 700 cm beyond its natural length.

→ We use the 1st part of the problem to find k. Since they mention work we need to use $W = \int_a^b kx\,dx$

$$1800\text{ J} = \int_0^3 kx\,dx$$

500 cm − 200 cm = 300 cm = 3 m

Put 0 if we start from the natural length

$$1800 = \frac{k}{2}x^2\Big|_0^3$$

$$1800 = \frac{k}{2}9$$

$$200(2) = k \qquad k = 400$$

→ Now that we have k we can address the 2nd part of the problem. They mention force so we use $F=kx$

$$F=400x$$

↖ how far from the natural length

700 cm beyond natural length = 7m

$$F=400(7)$$

$$\boxed{F=2800\,N}$$

$\boxed{\text{EX2}}$ A spring has a natural length of 10 in. An 800 lb force stretches the spring to 14 in.

a) Find how much work is done in stretching the spring from 10 in to 12 in

b) Find how much work is done in stretching the spring from 14 in to 16 in

c) How far beyond its natural length will a 1600 lb force stretch the spring?

→ As before we use the 1st piece of information to find k, but this time they give us force so we use

$F=kx$ ← distance from natural length

$800=k(\frac{1}{3})$

↖ 14 inches − 10 inches = 4 inches $=\frac{1}{3}$ ft

P3

* Remember if force is in lbs then distance is in feet and work is in ft-lbs
If force is in Newtons (N) then distance is in meters and work is in Nm (same as J)

$$K = 800(3) = 2400$$

→ Part a) mentions work so we use $W = \int_a^b kx\,dx$

$$W = \int_0^{\frac{2}{12}} 2400x\,dx$$

$12\text{in} - 10\text{in} = 2\text{in} = \frac{2}{12}\text{ft} = \frac{1}{6}\text{ft}$

starting at natural length

$$= \int_0^{1/6} 2400x\,dx = 1200x^2\Big|_0^{1/6}$$

$$= \boxed{1200\left(\tfrac{1}{6}\right)^2 \text{ ft-lb}}$$

→ part b) again asks for work

$$W = \int_a^b kx\,dx$$

$a = 14\text{inches} - 10\text{inches} = 4\text{inches} = \frac{1}{3}\text{ft}$

natural length

$b = 16\text{inches} - 10\text{inches} = 6\text{inches} = \frac{1}{2}\text{ft}$

$$W = \int_{1/3}^{1/2} 2400x\,dx = 1200x^2\Big|_{1/3}^{1/2}$$

$$= 1200\left[\tfrac{1}{4} - \tfrac{1}{9}\right] \text{ ft-lb}$$

→ c) mentions force so we use
$F = kx$

$$1600 = 2400x$$

$$\frac{2}{3} = \frac{1600}{2400} = x$$

$\boxed{\frac{2}{3}\text{ ft beyond natural length}}$

$\boxed{\text{EX 3}}$ Example 1 revisited use the natural length and spring constant from EX1 to find the work done stretching a spring from 500cm to 600cm.

natural length = 200cm
$k = 400$

$W = \int_a^b kx\,dx$ $\quad a = 500\text{cm} - 200\text{cm} = 300\text{cm} = 3\text{m}$

$= \int_3^4 400x$ $\quad b = 600\text{cm} - 200\text{cm} = 400\text{cm} = 4\text{m}$

$= 200x^2 \Big|_3^4$

$= 200[16 - 9]$

$= 200[7]$

$= 1400$ Nm or J