Calculus I Test 2 Version 1 No work=No credit!

1. (12 points)

a) State the definition of a derivative for a function f(x).

b) Use the definition of a derivative to find $f'(x)$ for $f(x)=\dfrac{x}{2x+1}$

2. (12 points) To receive full credit, justify your answers as we have done in class.

a) Find $\lim\limits_{x\to\infty}\dfrac{4-3e^x}{e^x+2}$

b) What type of asymptote have you found in part a)?

c) Find $\lim\limits_{x\to 2^-}\dfrac{x}{x-2}$

d) Find $\lim\limits_{x\to 2^+}\dfrac{x}{x-2}$

e) What type of asymptote is x=2?

3. (47 points) Find the derivatives of the following functions

a) $f(x)=\tan^{-1}(3x)+\log_3 x-e^3+x^\pi+\dfrac{1}{x^3}+\sqrt[4]{\ln(x)}+5^x$

b) $y=\dfrac{3x-5}{x^2+1}$

c) $g(x)=e^{2x}\sec(x)$

d) $y=\left(x^3-2x\right)^{\sqrt{x}}$

4. (10 points) Prove that $\dfrac{d}{dx}(\cot(x))=-\csc^2 x$

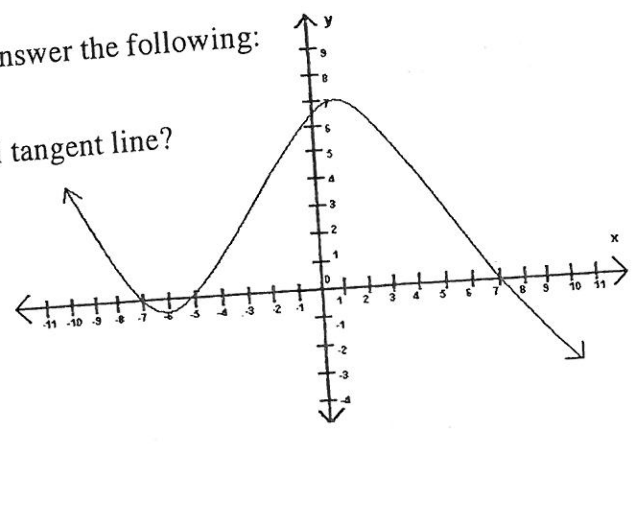
5. (10 points) Find the equation of the tangent line to $x^2+4xy+y^2=13$ at (2,1)

6. (3 points) There are 3 ways a function can fail to be differentiable at x=a. List 2 of them.

7. (6 points) Use the graph of f' given below to answer the following:

a) On what intervals is f increasing?

b) At what values of x does f have a horizontal tangent line?



C1 T2 V1 Solution

1. (12pts)

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

b) $f'(x) = \lim_{h \to 0} \frac{\frac{x+h}{2(x+h)+1} - \frac{x}{2x+1}}{h}$

$= \lim_{h \to 0} \frac{(x+h)(2x+1) - x(2x+2h+1)}{(2x+2h+1)(2x+1)h}$

$= \lim_{h \to 0} \frac{2x^2 + x + 2xh + h - 2x^2 - 2xh - x}{(2x+2h+1)(2x+1)h}$

$= \lim_{h \to 0} \frac{1}{(2x+2h+1)(2x+1)} = \frac{1}{(2x+1)^2}$

2. (12 points)

a) $\lim_{x \to \infty} \frac{4 - 3e^x}{e^x + 2} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}} = \lim_{x \to \infty} \frac{\frac{4}{e^x} - 3}{1 + \frac{2}{e^x}} = -3$

b) Horizontal

c) $\lim_{x \to 2^-} \frac{x}{x-2} = -\infty$

$\frac{1.9}{1.9-2}$

d) $\lim_{x \to 2^+} \frac{x}{x-2} = \infty$

$\frac{2.1}{2.1-2}$

e) Vertical

a) $f'(x) = \left(\frac{1}{1+(3x)^2}\right)\cdot 3 + \frac{1}{\ln(3)\,x} + 0 + \pi x^{\pi-1} - 3x^{-4} + \frac{1}{4}(\ln x)^{-3/4}\cdot\frac{1}{x}$

$+ \ln(5)\,5$

b). $y' = \frac{3(x^2+1) - (3x-5)(2x)}{(x^2+1)^2}$

$= \frac{3x^2+3-6x^2+10x}{(x^2+1)^2}$

$= \frac{-3x^2+10x+3}{(x^2+1)^2}$

c) $g'(x) = 2e^{2x}\sec(x) + e^{2x}\sec x \tan x$

d) $\ln y = \ln(x^3-2x)^{\sqrt{x}}$

$\ln y = \sqrt{x}\ln(x^3-2x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}x^{-1/2}\ln(x^3-2x) + \sqrt{x}\left(\frac{1}{x^3-2x}\right)(3x^2-2)$

$\frac{dy}{dx} = y\left(\frac{1}{2}x^{-1/2}\ln(x^3-2x) + \sqrt{x}\left(\frac{3x^2-2}{x^3-2x}\right)\right)$

$\frac{dy}{dx} = (x^3-2x)^{\sqrt{x}}\left[\frac{1}{2}x^{-1/2}\ln(x^3-2x) + \sqrt{x}\left(\frac{3x^2-2}{x^3-2x}\right)\right]$

4. (10 pts)

$$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} = -\csc^2 x$$ ✓

5. (10 pts) $2x + 4y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$(2,1)$ $4 + 4 + 8\frac{dy}{dx} + 2\frac{dy}{dx} = 0$

$$10\frac{dy}{dx} = -8$$

$$\frac{dy}{dx} = -\frac{8}{10} = -\frac{4}{5}$$

$$\boxed{y - 1 = -\frac{4}{5}(x-2)}$$

6. x is discontinuous at a
x has a corner at a
x has a vertical tangent at a

7. a) $(-\infty, 7)$ $(-5, 7)$

b) $x = -7$, $x = -5$, $x = 7$