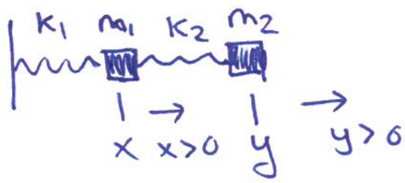
Coupled Mass Springs

☆ We can have multiple springs connected to masses. In the problems we'll look at we will have free oscillations – no external forces, no damping

To do these we will use $F_{net} = m \cdot a$ & Hooke's law, $F = kx$ ← spring constant $k > 0$

Ex 1



For the forces on m_1: $x(t)$ = position of the mass
x' = velocity
x'' = acceleration

$F_{net} = ma \longrightarrow F_{net} = m_1 x''$

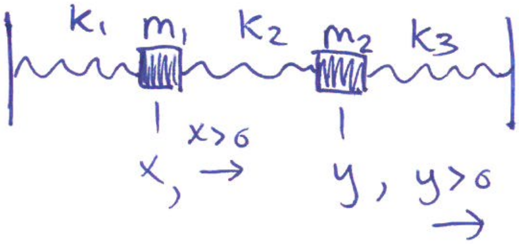
$$\boxed{m_1 x'' = -k_1 x + k_2(y-x)}$$

- If $x>0$ spring 1 stretches, so m_1 feels a pull to the left so we need a negative in front of $k_1 x$
- If $y-x>0$ spring 2 stretches, so m_1 feels a pull to the right so we should have $+k_2(y-x)$

$$\boxed{m_2 y'' = -k_2(y-x)}$$

- we can just do opposite of above or thinking about it, if $y-x>0$ spring 2 stretches so m_2 feels a pull to the left

Ex 2



As before we are just going to come up with the equations of motion

$$m_1 x'' = -k_1 x + k_2(y-x)$$

determining sign of k_1x

- $x>0$, m_1 feels a pull to the left (It helps if you really imagine this. Pretend you are actually stretching a spring)

determining sign of $k_2(y-x)$

- $y-x>0$, spring 2 is stretched, m_1 feels a pull to the right

$$m_2 y'' = -k_2(y-x) - k_3 y$$

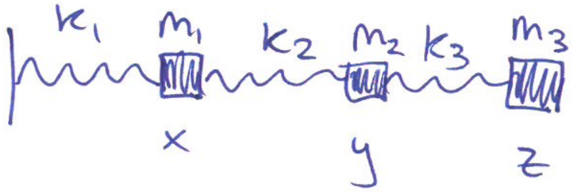
sign $k_2(y-x)$

- again you can do the opposite sign of above as a double check. If or think about it again $y-x>0$ Spring 2 is stretched so m_2 feels a pull to the left

sign of k_3y

- If $y>0$ spring 3 is compressed so m_2 feels a push to the left. You can also check for all of these if it still works (it should) for $y<0$. So in this case $y<0$ spring 3 is stretched & m_2 feels a pull to the right ✓

Ex 3



$$\underbrace{\boxed{m_1 x'' = -k_1 x + k_2(y-x)}}_{\text{Same as previous examples}}$$

$$\boxed{m_2 y'' = -k_2(y-x) + k_3(z-y)}$$

- If $z-y>0$ spring 3 is stretched so m_2 feels a pull to the right

$$\boxed{m_3 z'' = -k_3(z-y)}$$

- Again, opposite of above or think about it. $z-y>0$ spring 3 is stretched so m_3 feels a pull to the left