Finding derivatives using the definition

Ex. Polynomial

Technique: *Expand terms & distribute*

$f(x) = x^2 - 3x$

$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - [x^2 - 3x]}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$

Cancel terms

$= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h}$

Factor out an h & cancel

$= \lim_{h\to 0} \frac{\cancel{h}(2x + h - 3)}{\cancel{h}} = \lim_{h\to 0} 2x + h - 3$

Plug in zero for h

$= \boxed{2x - 3}$

Ex. Fractions

Technique: *Put terms in numerator over a common denominator*

$f(x) = \frac{1}{2x}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{2(x+h)} - \frac{1}{2x}}{h}$

$= \lim_{h\to 0} \frac{2x - (2x + 2h)}{2x(2x+2h)h}$

Cancel terms

$= \lim_{h\to 0} \frac{-2\cancel{h}}{2x(2x+2h)\cancel{h}} = \lim_{h\to 0} \frac{-2}{2x(2x+2h)} \boxed{= \frac{-2}{(2x)^2}}$

Ex. Square Roots

Technique: *Multiply by conjugate*

$f(x) = \sqrt{x+1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

* Cancel terms*

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+1} + \sqrt{x+1})}$$

* Cancel h's *

$$= \lim_{h \to 0} \frac{1}{(\sqrt{x+h+1} + \sqrt{x+1})}$$

* Plug in zero*

$$= \boxed{\frac{1}{2\sqrt{x+1}}}$$