

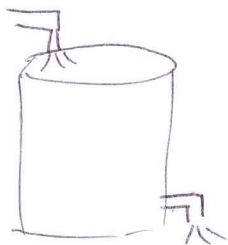
# MIXING PROBLEMS

①

$$\frac{dy}{dt} = r_i c_i - r_o c_o$$

P553

18. A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?



$$\begin{aligned}\frac{dy}{dt} &= r_i c_i - r_o c_o \\ &= \left(\frac{10 \text{ L}}{\text{min}}\right) \left(\frac{0.1 \text{ kg}}{\text{L}}\right) - \left(\frac{10 \text{ L}}{\text{min}}\right) \left(\frac{y(t)}{100 \text{ L}}\right) \\ &= 1 - \frac{y}{10}\end{aligned}$$

$$\int \frac{dy}{1 - \frac{y}{10}} = \int dt$$

$$\begin{aligned}u &= 1 - \frac{y}{10} \\ du &= -\frac{1}{10} dt\end{aligned}$$

$$-10 \int \frac{du}{u} = t + C$$

$$-10 du = dt$$

$$-10 \ln \left| 1 - \frac{y}{10} \right| = t + C$$

$$\ln \left| 1 - \frac{y}{10} \right| = -\frac{1}{10} t + C_2$$

$$\left| 1 - \frac{y}{10} \right| = e^{-\frac{1}{10} t + C_2} = e^{-\frac{1}{10} t} e^{C_2}$$

$$1 - \frac{y}{10} = K e^{-\frac{1}{10} t} \quad (K = \pm e^{C_2})$$

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$$1 - \frac{y}{10} = k e^{-\frac{1}{10}t}$$

$$10(1 - k e^{-\frac{1}{10}t}) = y$$

$$y(0) = 0 \quad (\text{Pure water has no salt})$$

$$\rightarrow 10(1 - k e^0) = 0$$

$$\text{or } k = 1$$

$$y = 10(1 - e^{-\frac{1}{10}t})$$

$$\boxed{y(6) = 10(1 - e^{-\frac{6}{10}})} \approx 4.52$$

p 520

37. A vat with 500 gallons of beer contains 4% alcohol by volume.

Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min, and the mixture is pumped out at the same rate. What is the percentage of alcohol after 1 hour?

$$\begin{aligned} \frac{dy}{dt} &= r_i c_i - r_o c_o \quad \text{where } y(t) = \text{amount of alcohol} \\ &= \left(\frac{5 \text{ gal}}{\text{min}}\right)(.06) - \left(\frac{5 \text{ gal}}{\text{min}}\right)\left(\frac{y(t)}{500 \text{ gal}}\right) \\ &= .3 - \frac{y}{100} \end{aligned}$$

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$$\frac{dy}{dt} = .3 - \frac{y}{100}$$

$$\int \frac{dy}{.3 - \frac{y}{100}} = \int dt$$

$$u = .3 - \frac{y}{100} \quad du = -\frac{1}{100} dy$$

$$-100 du = dy$$

$$\Rightarrow -100 \int \frac{1}{u} du = t + C$$

$$-100 \ln |.3 - \frac{y}{100}| = t + C$$

$$\ln |.3 - \frac{y}{100}| = -\frac{1}{100}t + C_2$$

$$|.3 - \frac{y}{100}| = e^{-\frac{1}{100}t + C_2} = e^{-\frac{1}{100}t} e^{C_2}$$

$$.3 - \frac{y}{100} = K e^{-\frac{1}{100}t}$$

$$100 (.3 - K e^{-\frac{1}{100}t}) = y$$

$$y(0) = .04(500) \left( \begin{array}{l} 4\% \text{ alcohol to start} \\ 500 \text{ gallons of beer} \end{array} \right) = 20$$

$$100 (.3 - K) = 20$$

$$\rightarrow K = .1$$

$$y = 30 - 10e^{-\frac{1}{100}t}$$

Amount of alcohol after 1 hr (60 min)

$$= y(1) = 30 - 10e^{-\frac{60}{100}} = 24.5$$

$$\frac{24.5}{500} = .049$$

$\rightarrow$  ~5% alcohol

④ Note:

If  $r_i \neq r_0$  then frequently the mixing problems will need to be solved using another method, which you will learn in Differential Equations.