Hydrostatic Force

In class we derived the formula:

top of plate

$$F = \int_a^b \rho \, L(y) \, (h-y) \, dy$$

depth

weight density

length of a cross section

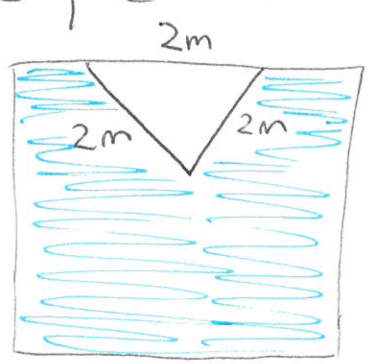
bottom of plate

☆ This set up is if the x-axis is placed at the bottom of the plate.

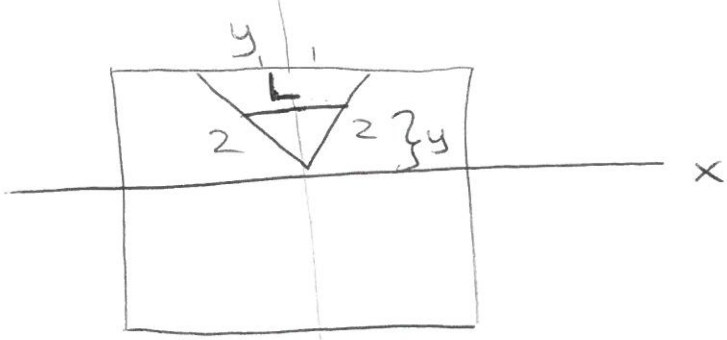
Examples from Calculus Books by Stewart, Briggs/Cochran/Gillett, & Larson/Hostetler/Edwards

Ex 1 Find the hydrostatic force on the submerged vertical plate

Given:



2



☆ We need the height of the triangle.

$2^2 = h^2 + 1 \qquad h = \sqrt{3}$

$$F = \int_a^b \rho \underbrace{L(y)}_{\text{Length of a cross section is a function of } y} (h-y)\, dy$$

Length of a cross section is a function of y

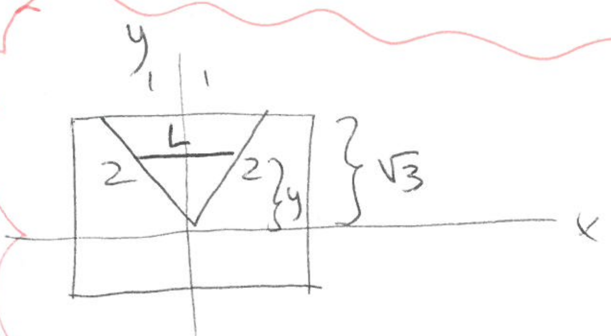
$$F = \int_0^{\sqrt{3}} 1000(9.8)\left[\frac{2}{\sqrt{3}}y\right](\sqrt{3}-y)\, dy \text{ N}$$

density of water

gravity

ρ would be given on a test

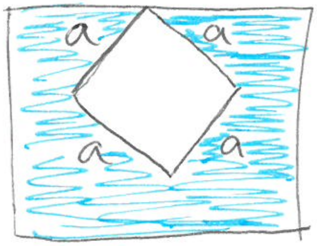
think how far your sample line is below the surface of the liquid.



We can find L with similar triangles (not the only way)

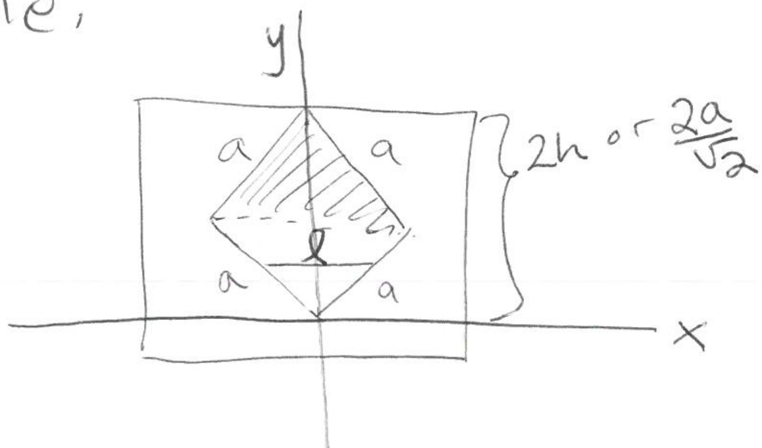
$$\frac{L}{y} = \frac{2}{\sqrt{3}} \rightarrow L = \frac{2}{\sqrt{3}}y$$

Ex2 Find the hydrostatic force on the submerged vertical plate

Given: 

☆ This problem is a little tricky. I'm going to approach this as 2 problems, basically top triangle & bottom triangle.

Bottom triangle:



height:

$a^2+a^2=(2h)^2$

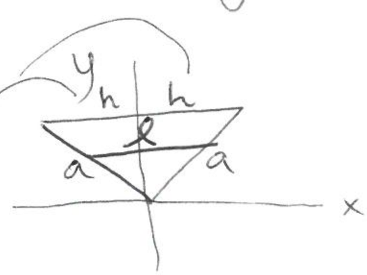
$2a^2=4h^2$

$h=\sqrt{\frac{a^2}{2}}=\frac{a}{\sqrt{2}}$

$$F_{\substack{\text{Bottom}\\\text{triangle}}}=\int_0^{\frac{a}{\sqrt{2}}}\rho\,(2y)\left(\frac{2a}{\sqrt{2}}-y\right)dy$$

we weren't given units

Diagonals of squares bisect each other

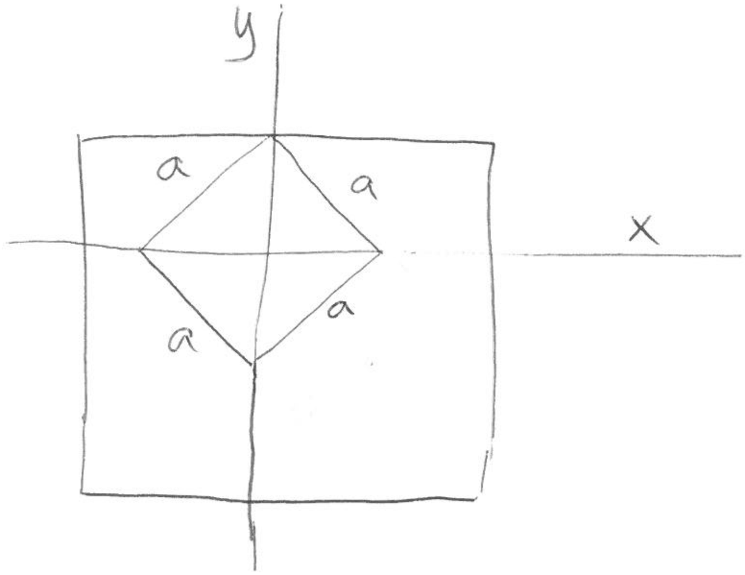


$\frac{\ell}{y}=\frac{2h}{h}=2$

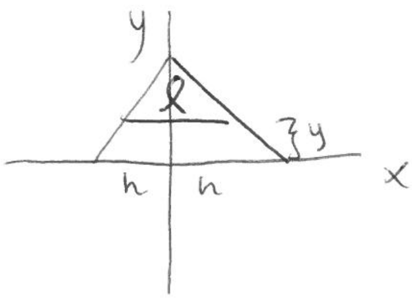
$\ell=2y$

4)

Top triangle!
I'm going to make my life easier & move the x-axis



$$F_{\substack{\text{Top}\\\text{triangle}}} = \int_0^{\frac{a}{\sqrt{2}}} \rho\, 2\left(\frac{a}{\sqrt{2}}-y\right)\left(\frac{a}{\sqrt{2}}-y\right)dy$$



$$\frac{l}{h-y} = \frac{2h}{h}$$

$$l = 2(h-y)$$

$h = \frac{a}{\sqrt{2}}$ so

$$l = 2\left(\frac{a}{\sqrt{2}}-y\right)$$

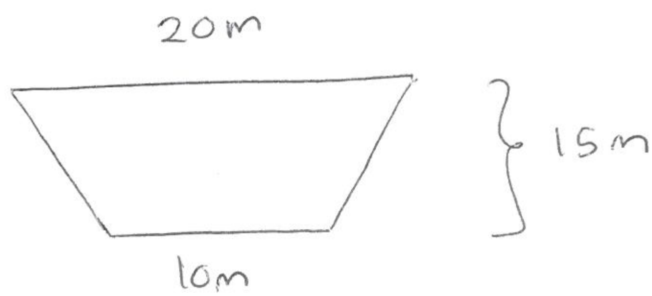
$$F = F_{B.T.} + F_{T.T.} = \int_0^{\frac{a}{\sqrt{2}}} \rho\,(2y)\left(\underbrace{\frac{2a}{\sqrt{2}}}_{\sqrt{2}a}-y\right)dy + \int_0^{\frac{a}{\sqrt{2}}} \rho\, 2\left(\frac{a}{\sqrt{2}}-y\right)^2 dy$$

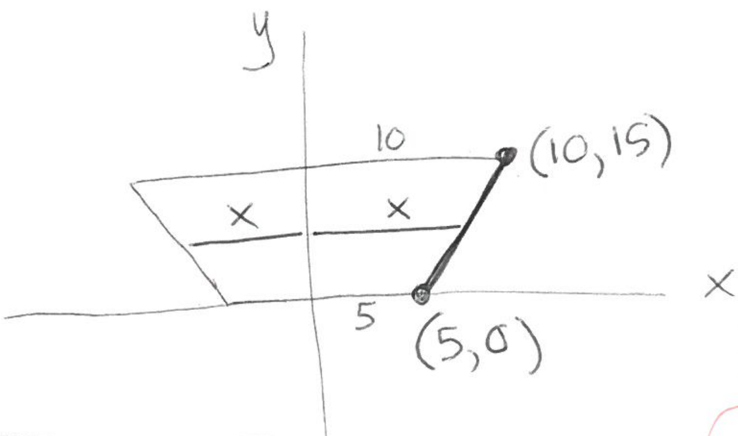
$$= \int_0^{\frac{a}{\sqrt{2}}} \rho\left(2\sqrt{2}ay - 2y^2\right)dy + \int_0^{\frac{a}{\sqrt{2}}} \rho\left(a^2 - 2\sqrt{2}ay + 2y^2\right)dy$$

$$= \int_0^{\frac{a}{\sqrt{2}}} \rho a^2\, dy = \rho a^2 y\Big|_0^{\frac{a}{\sqrt{2}}} = \boxed{\rho\frac{a^3}{\sqrt{2}}}$$

Ex 3 Assuming the water level is at the top of the dam, find the total force on the dam.

Given:





$$\boxed{F=\int_0^{15} 1000(9.8)\left[2\left(5+\frac{y}{3}\right)\right](15-y)\,dy \text{ N}}$$

$L=2x$, similar triangles aren't easily applied to this problem. Instead we are going to find the equation of the line & solve for x.

$$m=\frac{\Delta y}{\Delta x}=\frac{15-0}{10-5}=3$$

$y=mx+b \rightarrow y=3x+b$

(5,0) is on our line so

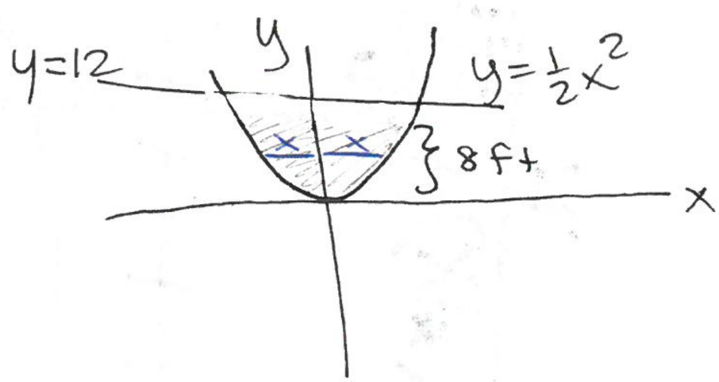
$0=3\cdot5+b \quad b=-15$

$y=3x-15 \rightarrow \frac{y+15}{3}=x$

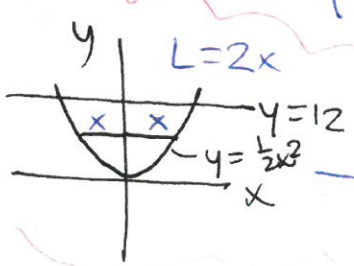
$x=5+\frac{y}{3}$

Ex 4

A large tank is designed with ends in the shape of the region between the curves $y=\frac{1}{2}x^2$ & $y=12$, measured in feet. Find the hydrostatic force on one end of the tank if it is filled to a depth of 8 ft with gasoline, $\rho = 42\ \text{lb/ft}^3$

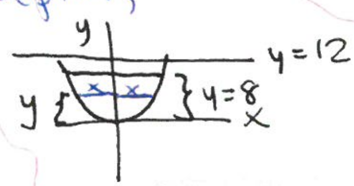


$$F = \int_0^8 \underbrace{42}_{\rho}\ \underbrace{2\sqrt{2y}}_{L(y)}\ \underbrace{(8-y)}_{\text{depth},}\ dy \text{ lb}$$



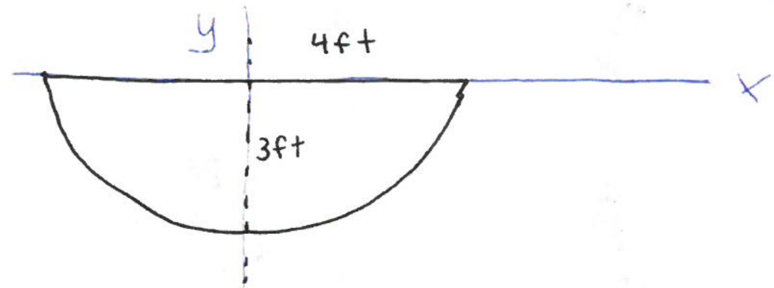
$\longrightarrow$ $2y = x^2$

$x = \sqrt{2y}$



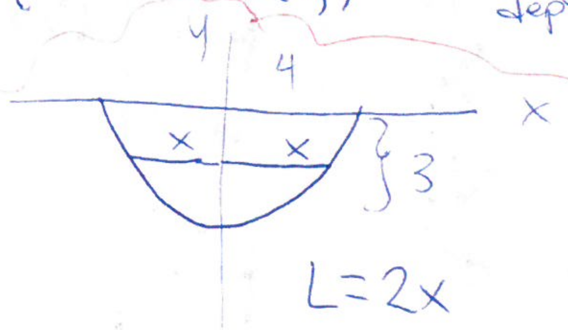
Ex 5 The figure is the vertical side of a form for poured concrete that weighs 140.7 lb/ft^3. Determine the force on this part of the concrete form.

Semiellipse $y = -\frac{3}{4}\sqrt{16-x^2}$



☆ This one would be harder if we move the x-axis, since the ellipse would also need to shift. Instead, I'll leave the x-axis where it is.

$$F = \int_{-3}^{0} \underbrace{140.7}_{\rho} \underbrace{\left[2\sqrt{16-\tfrac{16}{9}y^2}\right]}_{L(y)} \underbrace{(-y)}_{\text{depth}}\, dy \text{ lb}$$



$L = 2x$

$y = -\frac{3}{4}\sqrt{16-x^2}$

$-\frac{4}{3}y = \sqrt{16-x^2}$

$\frac{16}{9}y^2 = 16 - x^2$

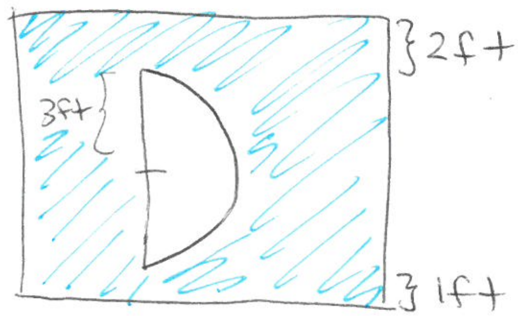
$x^2 = 16 - \frac{16}{9}y^2$

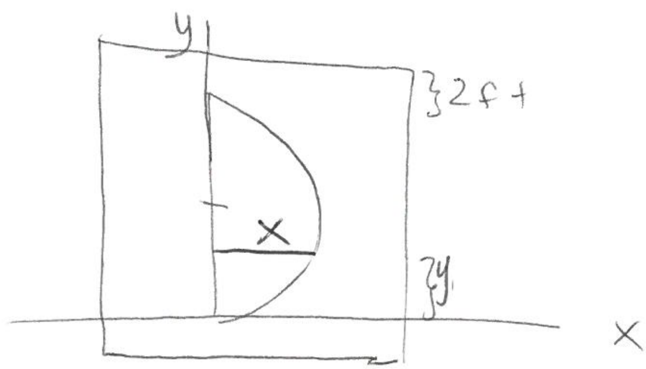
$x = \sqrt{16-\frac{16}{9}y^2}$

$x \geq 0$, since a distance

$\boxed{\text{Ex 6}}$ Find the hydrostatic force on the submerged vertical plate

Given:





$L(y) = x$, our circle is centered at $(0,3)$ so

$$(x-0)^2 + (y-3)^2 = 3^2$$

$$x = \sqrt{9-(y-3)^2}$$

$$\boxed{F = \int_0^6 62.4\left(\sqrt{9-(y-3)^2}\right)(8-y)\,dy \text{ lb.}}$$

ρ for H_2O, would be given

$6+2$