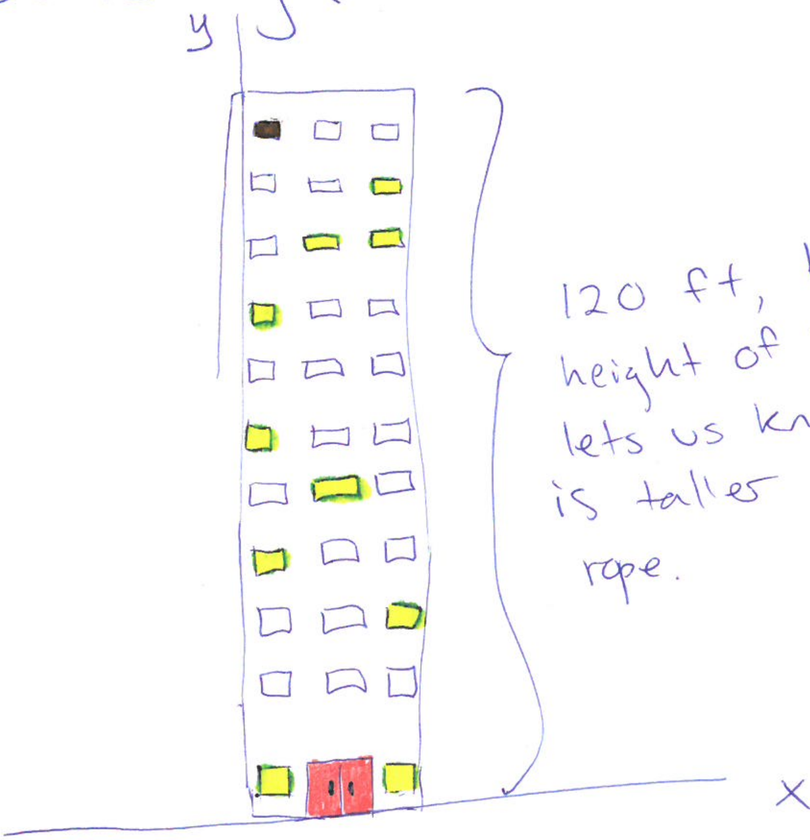
Work Lifting...

Examples from Calculus Books by Stewart, Briggs/Cochran/Gillett & Larson/Hostetler/Edwards

$\boxed{\text{Ex 1}}$ A heavy rope, 50 ft long, weighs 0.5 lb/ft & hangs over the edge of a building 120 ft high

a) How much work is done in pulling the rope to the top of the building?

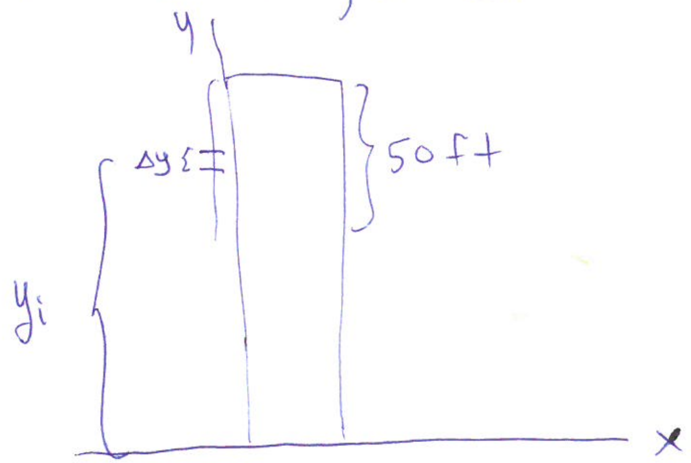


120 ft, knowing the height of the building lets us know the building is taller than the rope.

2

For this problem we'll use $W = Fd$, but we need to keep in mind that the top piece of rope barely needs to be moved while the bottom piece needs to move 50 ft.

The general strategy is to divide the rope into little pieces & figure out the work to move a little piece. We'll then add the work from lifting all the pieces together.



the x-axis doesn't have to be on the ground

$$W_{piece} \approx Fd = \underbrace{\left(.5 \tfrac{lb}{ft}\right)(\Delta y \text{ ft})}_{\text{weight of little piece of rope}} (120 - y_i)$$

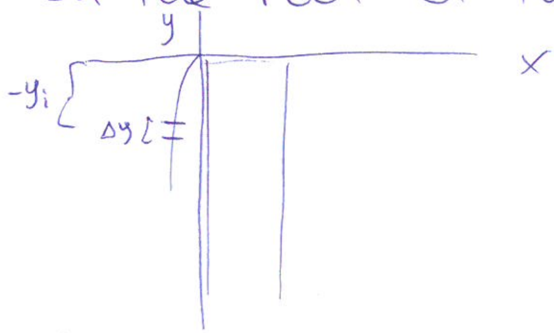
weight of little piece of rope where Δy = length of little piece & .5 lb/ft is the linear weight density

$$W_{\substack{\text{whole}\\ \text{rope}}} = \lim_{n\to\infty} \sum_{i=1}^{n} (.5)(120-y_i)\,\Delta y$$

$$W = \int_{70}^{120} (.5)(120-y)\,dy = .5\left(120y - \tfrac{1}{2}y^2\right)\Big|_{70}^{120}$$

$$= .5\left(120^2 - \tfrac{1}{2}\cdot 120^2 - 120\cdot 70 + \tfrac{1}{2}(70)^2\right) = \boxed{625 \text{ ft-lb}}$$

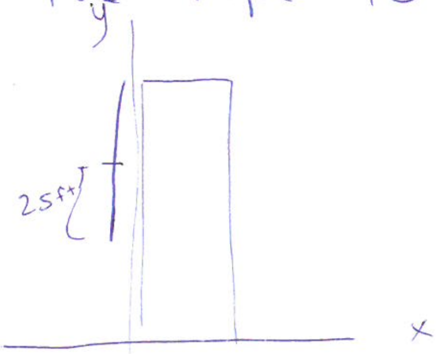
☆ This problem is easier with the x-axis on the roof of the building



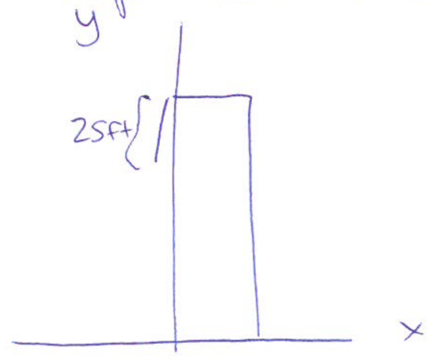
$$W = \int_{-50}^{0} .5(-y)\,dy = \boxed{625 \text{ ft-lb}}$$

Note this way we don't even need the height of the building.

b) How much work is done in pulling 1/2 the rope to the top of the building?



Before



Total Work =

$$\underbrace{\int_{95}^{120} (.5)(120-y)\,dy}_{\text{work to move top 25ft of rope to the top of the building}} + \underbrace{\left(.5\,\frac{lb}{ft}\right)(25\,ft)}_{\text{weight of bottom 25 ft of rope}}\underbrace{(25\,ft)}_{\text{distance rope moves}} = 468.75\ \text{ft-lb}$$

Note: the bottom 25 ft of rope uniformly moves up 25 ft.

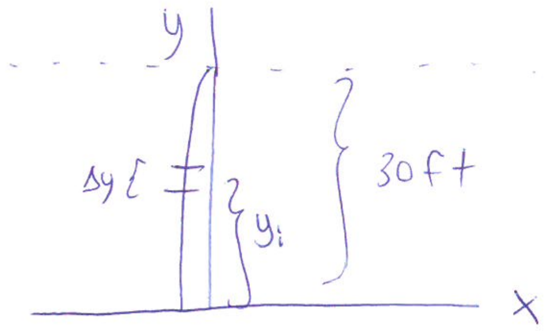
☆ If the x-axis is at the top of the building:

$$\int_{-25}^{0} .5(-y)\,dy + \frac{1}{2}\cdot 25^2$$

$\boxed{\text{Ex 2}}$ A 30m long chain hangs vertically from a cylinder attached to a winch. Assume there is no friction & that the chain has a density of 5kg/m

a) How much work is required to wind the entire chain onto the cylinder using the winch?

5



☆ This is similar to part a) of the previous problem. I'm going to change my set up so that the x-axis is at the bottom of the chain. I could also have done this with the previous problem.

$$W_{piece} \approx Fd = \underbrace{5\tfrac{kg}{m}(\Delta y\ m)\,9.8\tfrac{m}{s^2}}(30-y_i)$$

In this problem they gave us linear density (mass per unit length) instead of weight per unit length. To get weight per unit length we just multiply by gravity

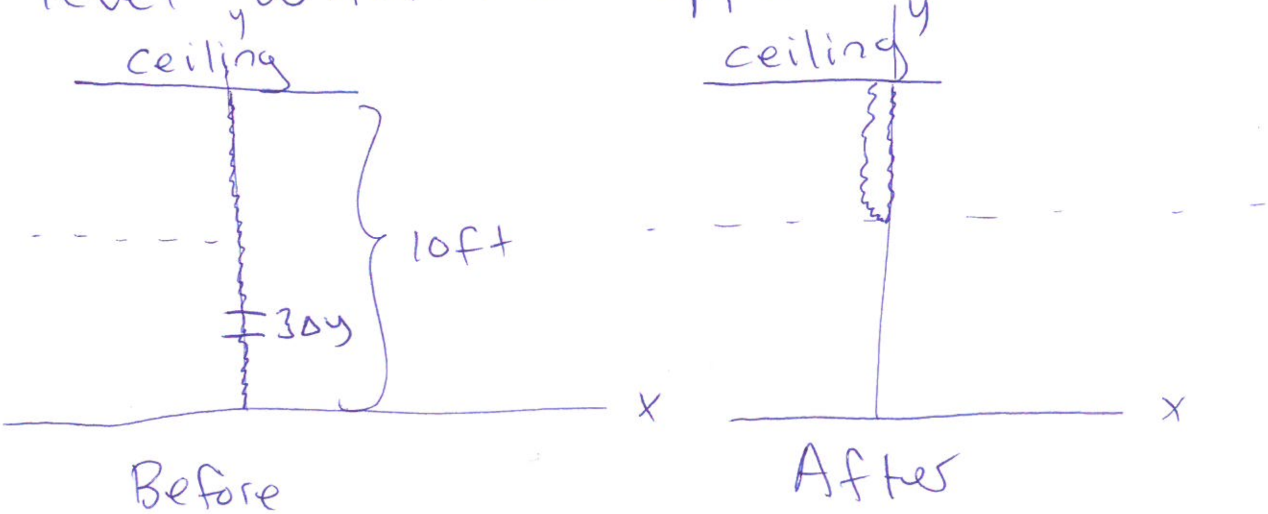
$$W_{total} = \int_0^{30} 5(9.8)(30-y)\,dy \text{ J}$$

b) How much work is required to wind the chain onto the cylinder if a 50 kg block is attached to the end of the chain?

$$W_{total} = W_{chain} + W_{block}$$

$$= \int_0^{30} 5(9.8)(30-y)\,dy + \underbrace{\underbrace{50(9.8)}_{\text{Weight}}\underbrace{(30)}_{d}}_{\text{Whole block is lifted 30m}} \text{ J}$$

$\boxed{\text{Ex3}}$ A 10ft chain weighs 25 lb & hangs from the ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end



7

Note: the top 1/2 of the chain doesn't move

linear weight density $= \frac{25\,lb}{10\,ft} = \frac{5}{2}\ lb/ft$

★ If they told us the mass of the rope is 25 kg & the total length was 10 m we would do $\frac{25(9.8)}{10}$

$$W_{piece} \approx Fd = \left(\frac{5}{2}\Delta y\right) \underbrace{2(5-y_i)}$$

we need that the piece at $y=0$ moves 10m & the piece at $y=5$ moves 0m

$$W_{total} = \int_0^5 \frac{5}{\cancel{2}} \cdot \cancel{2}(5-y)\,dy$$

$$= 25y - \frac{5y^2}{2}\Big|_0^5$$

$$= \frac{125}{2}\ ft\text{-}lb$$