MA 241 Test 2 Version 1 Show all of your work. No Calculators!

1. (14 points) Evaluate $\int 2\cos^3 x \sin^4 x dx$

2. (16 points) Evaluate $\int \frac{1}{x^2\sqrt{x^2-49}}dx$ Hint : $x = 7\sec\theta$

3. (12 points) Evaluate $\int_0^{\frac{\pi}{4}} 10\sec^4 x\, dx$

4. (15 points) Find $\int \frac{3x^3+27x+18}{x^2(x^2+9)}dx$

5. (11 points) Determine whether $\int_0^{\infty} e^{-3x}dx$ is convergent or divergent. If it converges, find its value. If it diverges, briefly explain why.

6. (14 points) Determine whether $\int_2^6 \frac{dx}{(x-2)^3}$ is convergent or divergent. If it converges, find its value. If it diverges, briefly explain why.

7. (18 points) a) Approximate $\int_1^{11} \sqrt{x^2+x}\, dx$ using Trapezoidal rule with $n = 5$.

You don't need to simplify after plugging your numbers into the square root.

b) Find the upper bound for the error estimate using $|E_T| \le \frac{K(b-a)^3}{12n^2}$,

where $|f''(x)| \le K$ for all x in [a,b]

Hint : $\mathbf{f''(x)} = \frac{-1}{(4x^2+4x)\sqrt{x^2+x}}$

C2T2V1 Solutions

1. (14 points) $\int 2\cos^3 x \sin^4 x \, dx$

$= \int 2\cos^2 x \sin^4 x \cos x \, dx$

$= \int 2(1-\sin^2 x)\sin^4 x \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int 2(1-u^2)u^4 \, du = \int 2u^4 - 2u^6 \, du$

$= \frac{2}{5}u^5 - \frac{2}{7}u^7 + C$

$= \boxed{\frac{2}{5}\sin^5 x - \frac{2}{7}\sin^7 x + C}$

2. (16 points) $x = 7\sec\theta \quad dx = 7\sec\theta\tan\theta \, d\theta$

$\int \frac{7\sec\theta\tan\theta \, d\theta}{7^2\sec^2\theta\sqrt{49\sec^2\theta - 49}}$

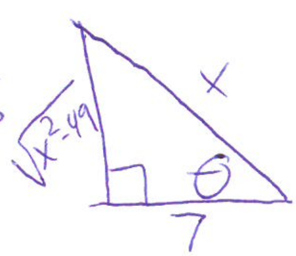
$= \int \frac{\cancel{7}\sec\theta\cancel{\tan\theta} \, d\theta}{7^2\sec^2\theta\cancel{\sqrt{49\tan^2\theta}}} = \int \frac{1}{49}\frac{1}{\sec\theta} \, d\theta$

$= \int \frac{1}{49}\cos\theta \, d\theta = \frac{1}{49}\sin\theta + C$

$= \boxed{\frac{1}{49}\frac{\sqrt{x^2-49}}{x} + C}$

$\frac{x}{7} = \sec\theta = \frac{1}{\cos\theta}$

$\frac{7}{x} = \cos\theta = \frac{adj}{hyp}$



3. (12 points)

$$\int_0^{\pi/4} 10\sec^2 x \sec^2 x \, dx$$

$$= \int_0^{\pi/4} 10(\tan^2 x + 1)\sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int_0^1 10(u^2+1)\,du$$

$$= 10\left(\tfrac{1}{3}u^3 + u\right)\Big|_0^1 = 10\left(\tfrac{1}{3}+1\right) = \frac{40}{3}$$

4. (15 points) $\displaystyle\int \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}\,dx$

$$Ax(x^2+9) + B(x^2+9) + (Cx+D)x^2 = 3x^3 + 27x + 18$$

$$Ax^3 + 9Ax + Bx^2 + 9B + Cx^3 + Dx^2 =$$

$A+C=3$ $\quad B+D=0$ $\quad 9A=27$ $\quad 9B=18$

$A=3$ $\quad B=2$ $\quad D=-2$

$C=0$

$$= \int \frac{3}{x} + \frac{2}{x^2} - \frac{2}{x^2+9}\,dx$$

$$= \boxed{3\ln|x| - 2/x - \tfrac{2}{3}\tan^{-1}\left(\tfrac{x}{3}\right) + C}$$

5. (11 points)

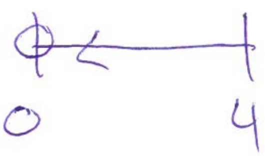
$$\int_0^\infty e^{-3x}\,dx = \lim_{t\to\infty} \int_0^t e^{-3x}\,dx$$

$$= \lim_{t\to\infty} -\frac{1}{3}e^{-3x}\Big|_0^t = \lim_{t\to\infty} -\frac{1}{3}e^{-3t} + \frac{1}{3}e^{0}$$

$$= \boxed{\frac{1}{3}}$$ converges

6. (14 points $\displaystyle\int_2^6 \frac{dx}{(x-2)^3}$ $\quad u = x-2$, $du = dx$

$$= \int_0^4 \frac{du}{u^3} = \lim_{t\to 0^+} \int_t^4 u^{-3}\,du$$

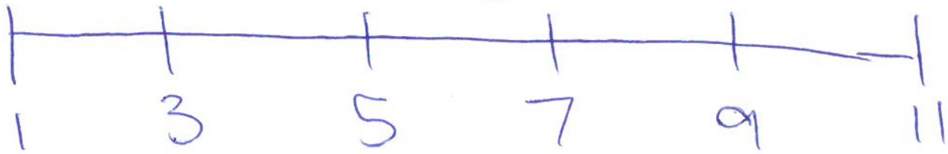


$$= \lim_{t\to 0^+} -\frac{1}{2}u^{-2}\Big|_t^4 = \lim_{t\to 0^+} -\frac{1}{2}\cdot 4^{-2} + \frac{1}{2}t^{-2} \to \infty$$

diverges

7. (18 points)

a) $\Delta x = \frac{b-a}{n} = \frac{11-1}{5} = 2$



$$T_5 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

$$= \frac{2}{2}\left[\sqrt{1^2+1} + 2\sqrt{3^2+3} + 2\sqrt{5^2+5} + 2\sqrt{7^2+7} + 2\sqrt{9^2+9} + \sqrt{11^2+11}\right]$$

b) $$|E_T| \le \frac{\frac{1}{(4+4)\sqrt{1+1}}(11-1)^3}{12 \cdot 5^2}$$