

MA 242 - 050 Test 2 Version 1

1. (16 points) Classify all critical points of $f(x,y) = x^4 + 2y^2 - 4xy$

Fully justify your answers as we have done in class.

2. (18 points)

- a) Find the directional derivative of $f(x,y) = \ln(2x + y^2)$ at $P(2,1)$ in the direction of $Q(3,4)$
- b) Find the direction of the maximum rate of change of $f(x,y) = \ln(2x + y^2)$ at $P(2,1)$
- c) For *any function f* determine when the directional derivative of f is half its maximum value
Show your work and explain your answer

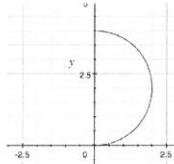
3. (13 points)

- a) Find $\lim_{(x,y) \rightarrow (0,0)} \frac{5xy^4}{2x^2 + y^8}$ if it exists or show that the limit does not exist. Justify your work
- b) If $\lim_{(x,y) \rightarrow (1,2)} f(x,y) = 7$, what if anything, can be said about $f(1,2)$?

4. (13 points) The length (l), width (w), and height (h) of a box change with respect to time. At the instant when the $w = h = 2$ m, $l = 4$ m, the l and w are increasing at a rate of 2 m/s, while h is decreasing at a rate of 1 m/s. Use the Chain Rule to write out the formula for $\frac{dV}{dt}$ and then find it at this instant.

5. (32 points) Use $f(x,y) = x^2 - y^2$ to answer the following:

- a) Find the global maximum and minimum values of $f(x,y) = x^2 - y^2$ on the region D where D is bounded by $x = \sqrt{4 - (y-2)^2}$ and the y-axis. Fully justify your answers as we have done in class. Label your work.



- b) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 = 16$

6. (8 points) Find the first partial derivatives of $z = e^{x/y}$

C3 T2 Solutions

1. (16 points)

$$f_x = 4x^3 - 4y = 0 \quad 4x(x^2 - 1) = 0 \quad x=0 \\ x=\pm 1$$

$$f_y = 4y - 4x = 0 \quad y=x$$

$$f_{xx} = 12x^2$$

$$f_{xy} = -4$$

$$f_{yy} = 4$$

$$D = 48x^2 - 16$$

$$D(0,0) = -16 < 0 \rightarrow (0,0) \text{ is a saddle pt}$$

$$D(1,1) = 48 - 16 > 0$$

$$f_{xx}(1,1) = 12 > 0 \rightarrow (1,1) \text{ is a local min}$$

$$D(-1,-1) = 48 - 16 > 0$$

$$f_{xx}(-1,-1) = 12 > 0 \quad (-1,-1) \text{ local min}$$

2. (18 points)

a) $\nabla f = \left\langle \frac{2}{2x+y^2}, \frac{2y}{2x+y^2} \right\rangle$

$$\nabla f(2,1) = \left\langle \frac{2}{5}, \frac{2}{5} \right\rangle$$

$$\vec{PQ} = \langle 1, 3 \rangle$$

$$\hat{u} = \left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$$

$$D_{\hat{u}} f = \left\langle \frac{2}{5}, \frac{2}{5} \right\rangle \cdot \left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle = \frac{2}{5\sqrt{10}} + \frac{6}{5\sqrt{10}} = \boxed{\frac{8}{5\sqrt{10}}}$$

b) $\left\langle \frac{2}{5}, \frac{2}{5} \right\rangle$

c) $D_{\hat{u}} f = \nabla f \cdot \hat{u} = \|\nabla f\| \|\hat{u}\| \cos \theta = \|\nabla f\| \cos \theta = \frac{1}{2} \|\nabla f\|$

$$\cos \theta = \frac{1}{2} \quad \theta = \frac{\pi}{3}$$

When the angle between ∇f & \hat{u} is $\frac{\pi}{3}$

3. (13 points)

a) $x=0 \lim_{(0,y) \rightarrow (0,0)} \frac{0}{y^8} = 0$

$x=y^4 \lim_{(y^4, y) \rightarrow (0,0)} \frac{5y^4 y^4}{2y^8 + y^8} = \frac{5}{3}$

$0 \neq \frac{5}{3}$ limit DNE

b) If f is continuous $f(1,2)=7$. otherwise it is not (that is $f(1,2) \neq 7$).

4. (13 points)

$$V=lwh$$

$$\frac{dV}{dt} = V_l \frac{dl}{dt} + V_w \frac{dw}{dt} + V_h \frac{dh}{dt}$$

$$= wh \frac{dl}{dt} + lh \frac{dw}{dt} + lw \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2 \cdot 2 \cdot 2 + 4 \cdot 2 \cdot 2 + 4 \cdot 2 \cdot (-1)$$

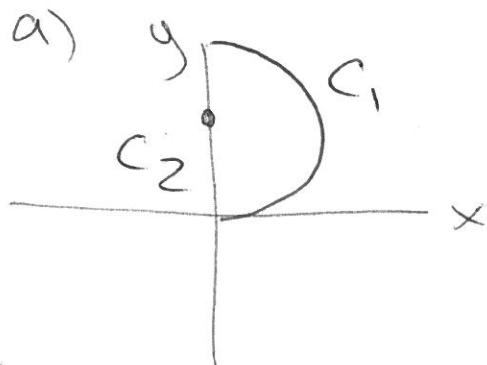
$$= 16$$

6. (8 pts)

$$\frac{\partial z}{\partial x} = \frac{1}{y} e^{x/y}$$

$$\frac{\partial z}{\partial y} = \frac{-x}{y^2} e^{x/y}$$

S. (32 points)



$$\begin{aligned}f_x &= 2x = 0 \\f_y &= -2y = 0\end{aligned}$$

critical pt (0,0)

C_1

$$x = \sqrt{4 - (y-2)^2}$$

$$f(\sqrt{4 - (y-2)^2}, y) = 4 - (y-2)^2 - y^2$$

$$f_y(\sqrt{4 - (y-2)^2}, y) = -2(y-2) - 2y = 0$$

$$\begin{aligned}-4y + 4 &= 0 \\y &= 1 \quad x = \sqrt{3}\end{aligned}$$

C_2

$$x = 0$$

$$f(0, y) = -y^2$$

$$f_y(0, y) = -2y = 0$$

b). $\nabla f = \lambda \nabla g$

$$\langle 2x, -2y \rangle = \lambda \langle 2x, 2y \rangle$$

$$2x = \lambda 2x \quad x = 0 \quad \text{or} \quad \lambda = 1$$

$$-2y = 2y\lambda \quad y = 0 \quad \text{or} \quad \lambda = -1$$

$$\text{if } x = 0, \quad y = \pm 4 \quad f(0, 4) = -16 \quad \left. \begin{array}{l} \min \\ f(0, -4) = -16 \end{array} \right\}$$

$$\text{if } \lambda = 1 \rightarrow y = 0 \rightarrow x = \pm 4 \quad f(\pm 4, 0) = 16 \quad \left. \begin{array}{l} \max \end{array} \right\}$$

(candidates)

critical pt

$$f(0, 0) = 0$$

border

$$f(\sqrt{3}, 1) = 2$$

$$\left. \begin{array}{l} f(0, 0) = 0 \end{array} \right\}$$

ext pts

$$f(0, 4) = -16$$

$$f(0, 0) = 0$$

Global min
value = -16

Global max

value = 2