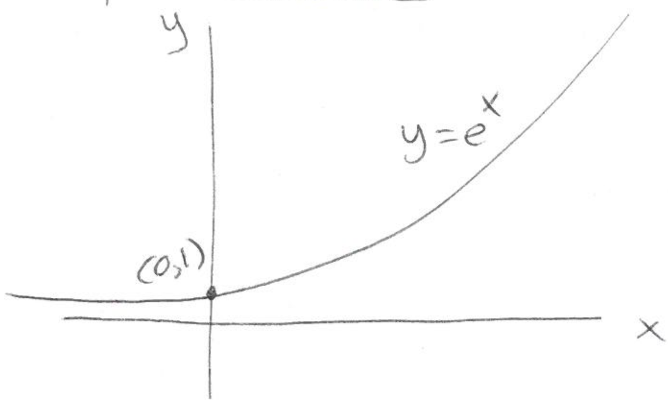
Precalc & Calc Refresher

This is *not* a comprehensive list of every topic you should know from precalculus & calculus 1, but this should hopefully cover enough :)

Exponentials:

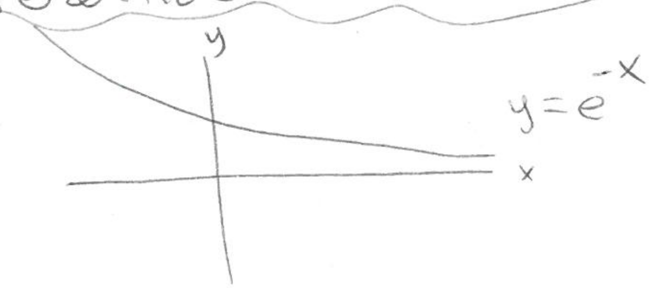


$y=e^x$ Quick Facts:

- $\lim_{x\to\infty} e^x \to \infty$
- $\lim_{x\to-\infty} e^x = 0$
- $e^0=1$
- $e^{a+b}=e^a e^b$
- $(e^a)^b = e^{ab}$
- Domain: x can be all reals!
- Range: $y>0$

$e=2.718....$
Although we can have a^x where a is a number other than e, we really don't see that a lot in Calc 2, Calc 3, or Differential Equations.

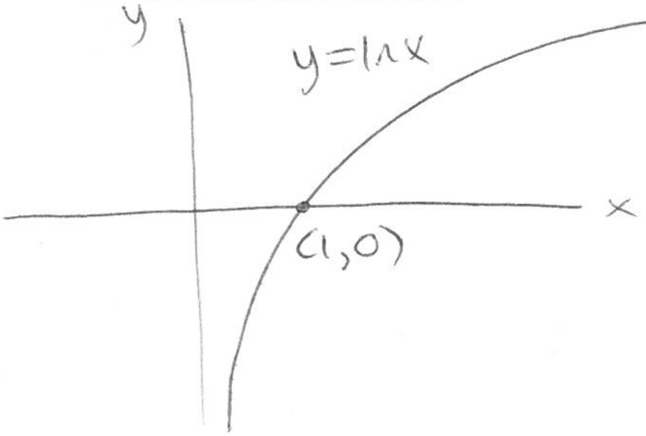
FYI:



$\lim_{x\to\infty} e^{-x} = 0$ (we can see this from the graph or by a chart)

x	e^{-x}
10	$e^{-10}=\frac{1}{e^{10}}$
100	$e^{-100}=\frac{1}{e^{100}}$

Logarithms:



Quick Facts:

$\lim_{x\to\infty} \ln x \to \infty$

$\lim_{x\to 0^+} \ln x \to -\infty$

This means we approach 0 on the right, that means numbers greater than zero

$\ln 1 = 0$ ($y=\ln x$ & $y=e^x$ are inverse functions so $y=\ln x \Leftrightarrow x=e^y$
$y=\ln 1 \Leftrightarrow 1=e^y$
$\to y=0$)

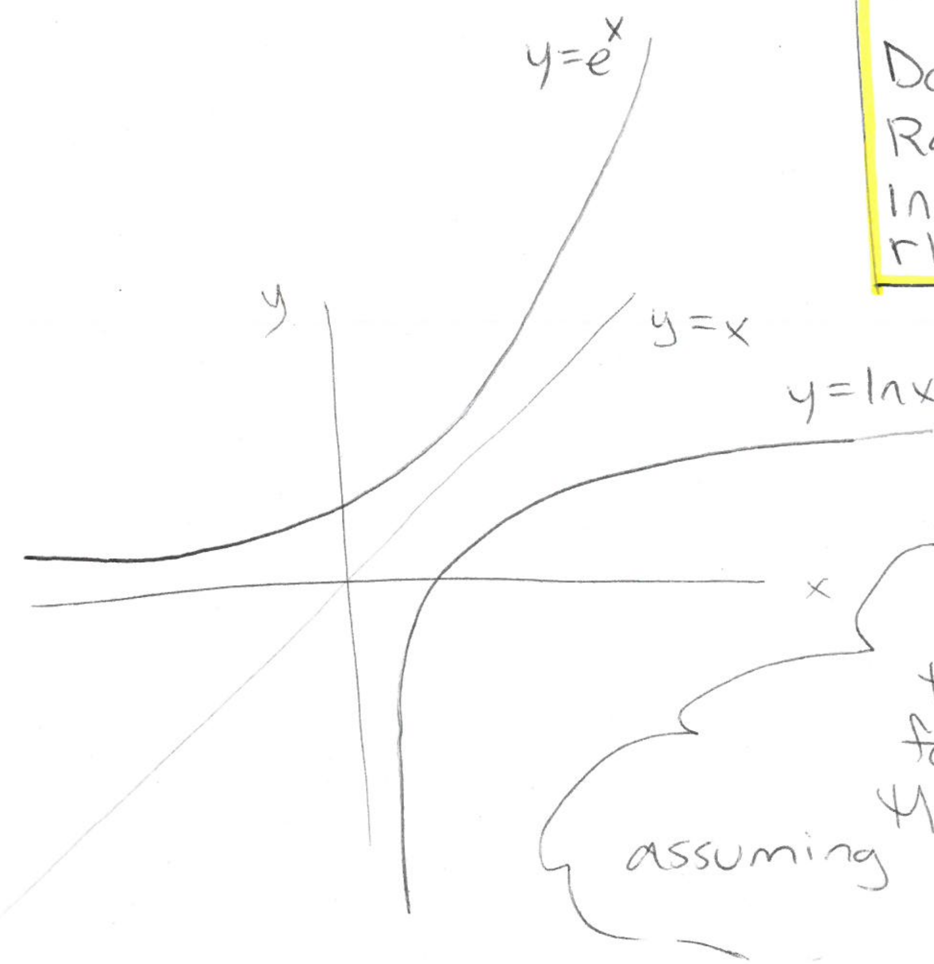
$\ln(e^x) = x$ ($\ln e = \ln e^1 = 1$
$\ln(e^{f(x)}) = f(x)$)

Domain: $x>0$

Range: All reals

$\ln(ab) = \ln a + \ln b$

$r\ln x = \ln(x^r)$

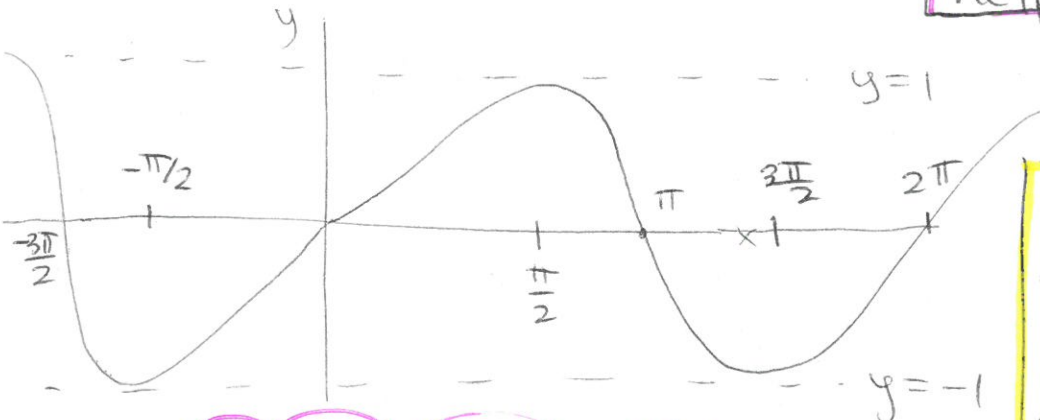


Inverse functions are symmetric about the line $y=x$. Imagine folding the graph there & they'll match up assuming you were careful

Trigonometry

See the trig basics worksheet under Test 1 Resources for more help with this!!

$y = \sin x$



Quick Facts:

Domain: all reals

Range: $-1 \le y \le 1$

$\lim_{x \to \infty} \sin x$ DNE

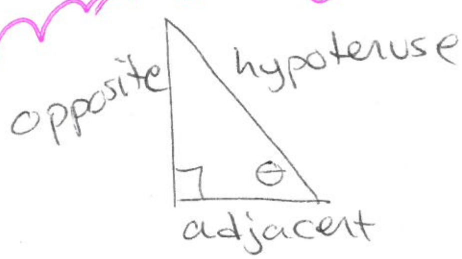
$y(n\pi) = \sin n\pi = 0$ for $n = 0, \pm 1, \pm 2, \ldots$

$y(\pi/2) = \sin \frac{\pi}{2} = 1$

$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

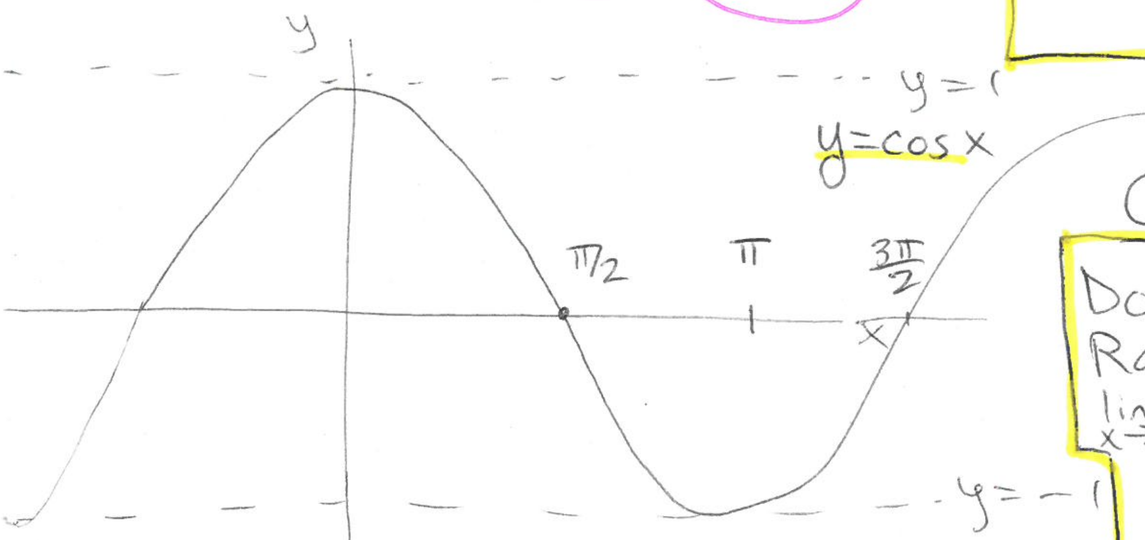
$\sin \frac{\pi}{6} = \frac{1}{2}$



$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$y = \cos x$



Quick Facts!

Domain: all reals

Range: $-1 \le y \le 1$

$\lim_{x \to \infty} \cos x$ DNE

$\cos 0 = 1$

$\cos \frac{\pi}{2} = 0$

$\cos \frac{3\pi}{2} = 0$

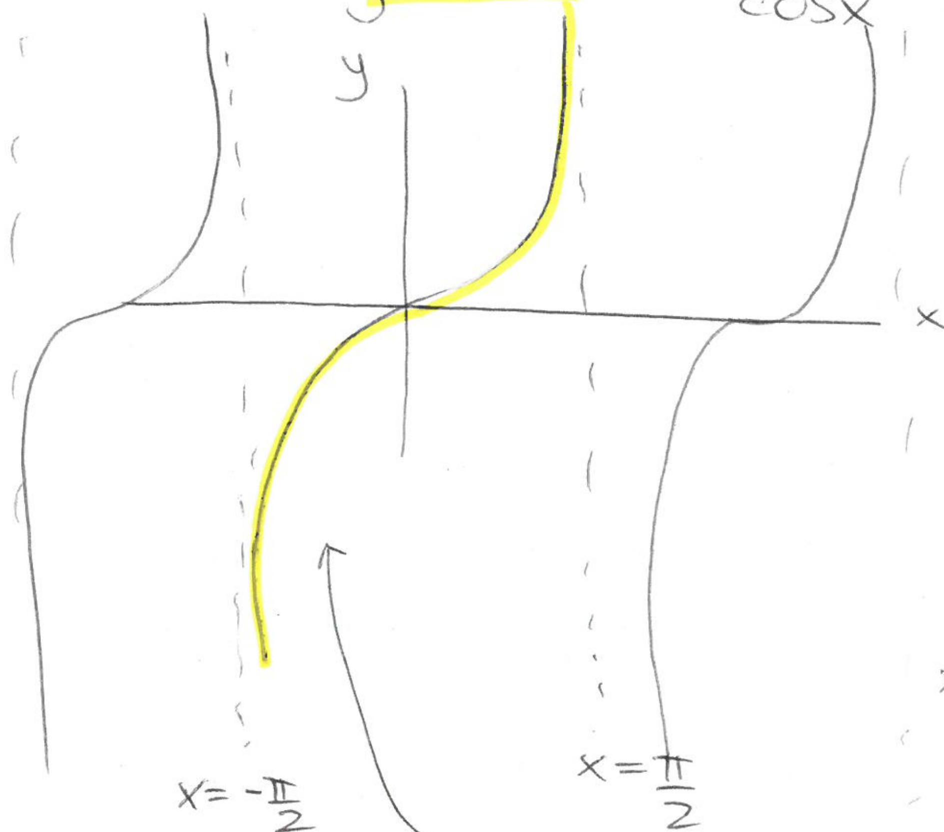
$\cos 2\pi = 1$

$\cos \pi/4 = \frac{\sqrt{2}}{2}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$y = \tan x = \dfrac{\sin x}{\cos x} = \dfrac{\text{opp}}{\text{adj}}$



Sometimes we just focus on this one.

Quick Facts

Domain: can't divide by zero so any x that makes $\cos x =$ is not included

$x \neq \dfrac{(2n+1)\pi}{2}$ where $n = 0, \pm 1, \pm 2$

Range: all reals

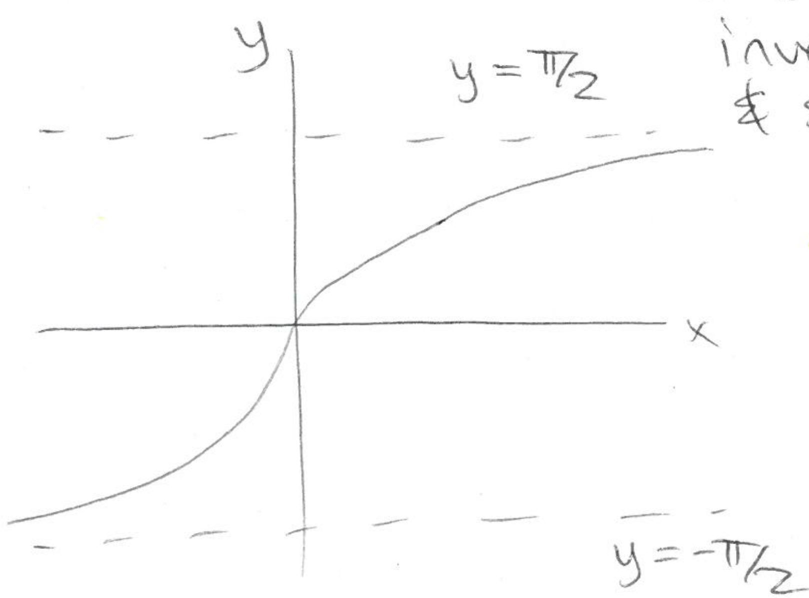
$\lim_{x \to \frac{\pi}{2}^-} \tan x \to \infty$

$\lim_{x \to \left(-\frac{\pi}{2}\right)^+} \tan x \to -\infty$

$\tan 0 = 0$

$\tan \pi/4 = 1$

$y = \tan^{-1} x$ (Also called arctangent -- inverse function of tangent, like $\cos^{-1} x$ & $\sin^{-1} x$ are inverse functions of cosine & sine respectively)



Quick Facts

Domain: all reals

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$

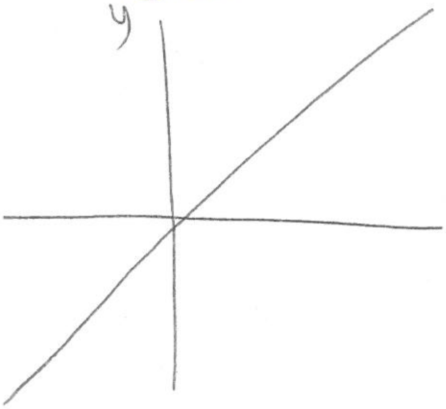
$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

$\tan^{-1} 0 = 0$

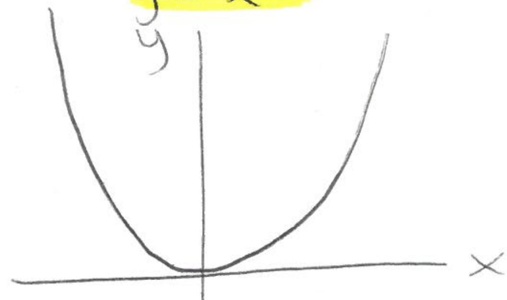
$\tan^{-1} \frac{\pi}{4} = 1$

5 Other Basic Graphs

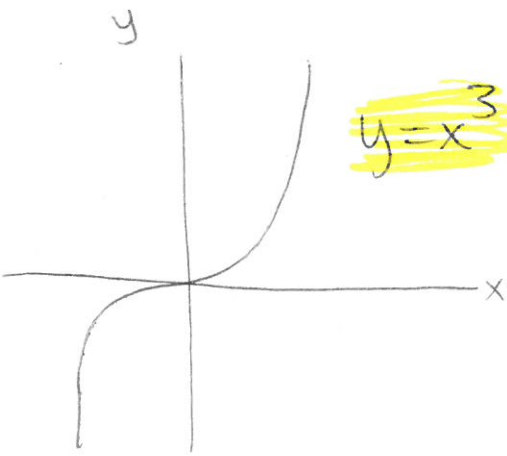
$y=x$



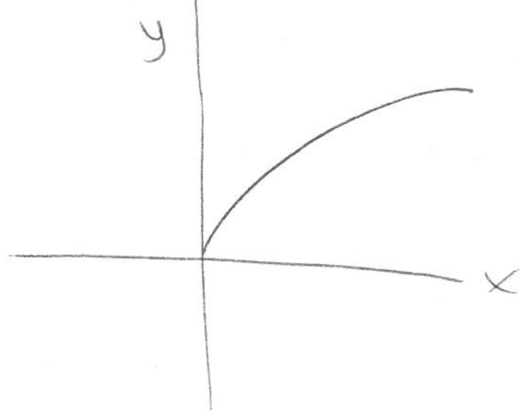
$y=x^2$



$y=x^3$

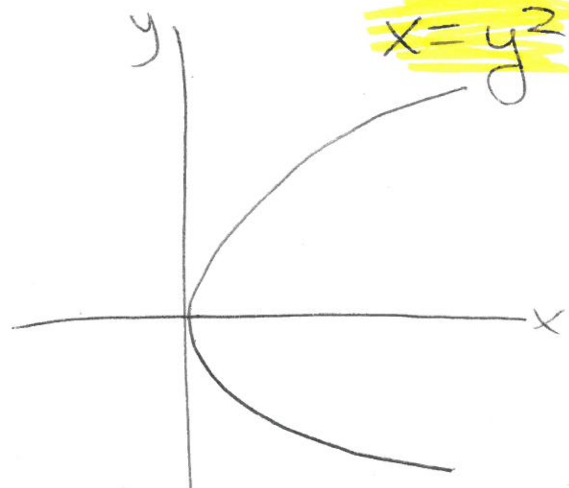


$y=\sqrt{x}$



$y=x^4$

$x=y^2$



$y={}^{1}/_{x}$

$\lim_{x\to 0^+} \frac{1}{x} \to \infty$

$\lim_{x\to \infty} \frac{1}{x} = 0$

$y=\frac{1}{x^2}$

$\lim_{x\to 0^+} \frac{1}{x^2} \to \infty$

$\lim_{x\to \infty} \frac{1}{x^2} = 0$

Derivatives (Not every derivative ever!)

$\frac{d}{dx}(c) = 0$ derivative of a constant = 0

$\boxed{\text{ex } \frac{d}{dx}(57) = 0}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(x^n) = nx^{n-1}$ Power rule

$\boxed{\text{ex } \frac{d}{dx}\left[x^{\frac{1}{7}}\right] = \frac{1}{7}x^{-\frac{6}{7}}}$

$\frac{d}{dx}[\underset{f(x)}{f}\,\underset{g(x)}{g}] = f'g + fg'$ Product Rule

ex $\frac{d}{dx}[x^2e^x] = 2xe^x + x^2e^x$

$\frac{d}{dx}[e^x] = e^x$!

$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{f'g - fg'}{g^2}$ Quotient Rule

$\frac{d}{dx}\left[\frac{\ln x}{\sqrt{x}}\right] = \frac{d}{dx}\left[\frac{\ln x}{x^{1/2}}\right]$

$= \frac{(1/x)\,x^{1/2} - (\ln x)\,\frac{1}{2}x^{-1/2}}{(x^{1/2})^2}$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[f(g(x))] = f'(g(x))\,g'(x)$ Chain Rule

Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$$

We use this all the time. If you need help with the Calc 3 version of the Chain Rule for $z = f(x,y)$ where x & y are functions of other variables see the worksheet on that on both my Calc 3 & MA 401 pages

ex $$\frac{d}{dx}\left[\tan(f(x))\right] = \underbrace{\sec^2}_{\substack{\text{derivative}\\ \text{of the}\\ \text{outside}\\ \text{function}}}\underbrace{(f(x))}_{\substack{\text{inside}\\ \text{function}\\ \text{stays the}\\ \text{same}}}\ \underbrace{f'(x)}_{\substack{\text{derivative of}\\ \text{the}\\ \text{inside}\\ \text{function}}}$$

ex $$\frac{d}{dx}\left[\tan(\ln x)\right] = \sec^2(\ln x)\,\frac{1}{x}$$

ex $$\frac{d}{dx}\left[\underbrace{e^{2x}}_{f}\,\underbrace{\cos 3x}_{g}\right] = \underbrace{2e^{2x}}_{f'}\,\underbrace{\cos 3x}_{g} + \underbrace{e^{2x}}_{f}\underbrace{(-3\sin 3x)}_{g'}$$

chain rule & product rule

inside functions are $2x$ for e & $3x$ for $\cos$.

ex $$\frac{d}{dx}\left[\sin x^3\right] = \cos(x^3)\,3x^2$$

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$$ (Not chain rule yet, I just forgot to put it)

ex $$\frac{d}{dx}\left[\tan^{-1}\left(x^{1/3} - \underbrace{\tfrac{1}{x}}_{-x^{-1}} + 8\right)\right] = \frac{1}{1+\left(x^{1/3}-\frac{1}{x}+8\right)^2}\cdot\left(\frac{1}{3}x^{-2/3} + \frac{1}{x^2} + 0\right)$$

Integration

Undo a derivative

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$$

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Evaluation theorem
Also part of the Fundamental theorem of Calculus

U-substitution

There is a whole worksheet on this if one example isn't enough

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

If $u = g(x)$
$du = g'(x)dx$

Ex $\int \cos x \sin^7 x\,dx$ $u = \sin x$
$du = \cos x\,dx$

$$= \int u^7\,du = \frac{1}{8}u^8 + C$$
$$= \frac{1}{8}\sin^8 x + C$$

8.5)

I mention this on the very long & very helpful u-substitution worksheet but for Calc 2, Calc 3, etc students just memorize the following:

$$\boxed{\begin{aligned}
&\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C \\
&\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C \\
&\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C \\
&\int \frac{1}{x+a}\,dx = \ln|x+a| + C \\
&\int (x+a)^n\,dx \quad (n \neq -1) = \frac{(x+a)^{n+1}}{n+1} + C
\end{aligned}}$$

w/ u-sub

Ex $\int \frac{1}{x-6}\,dx \quad u = x-6 \quad du = dx$

$$= \int \frac{1}{u}\,du = \ln|u| + C = \ln|x-6| + C$$

or

$$\int \frac{1}{x-6}\,dx = \ln|x-6| + C$$

Both results are based off a u-sub but one is shown explicitly.

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Integration by Parts

If you need a lot of help on this please do yourself a kindness & check out the Integration by Parts worksheet under Test 1 resources

$$\int f(x)\,g'(x)\,dx = fg - \int g f'\,dx$$

$$\int u\,dv = uv - \int v\,du$$

Pick u by LIATE (Logs, Inverse trig, Algebraic, Trig, Exponential) then dV is everything left over.

Common Forms: Not all forms! Usually integration by parts is one of these

$$\boxed{\begin{array}{l}\int x^n \cos x\,dx \\ \int x^n \sin x\,dx \\ \int x^n e^x\,dx \\ \int x^n \ln x\,dx\end{array}}$$

Ex $\displaystyle\int_0^1 \frac{(y+1)}{e^{2y}}\,dy = \int_0^1 (y+1)e^{-2y}\,dy$

$u = y+1$ $\quad V = -\frac{1}{2}e^{-2y}$ ← using the result of $\int e^{ax}dx = \frac{1}{a}e^{ax} + C$

$du = dy$ $\quad dV = e^{-2y}$

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$$\int_0^1 (y+1)e^{-2y}dy = uv - \int v\,du$$
$$= (y+1)\left(-\tfrac{1}{2}e^{-2y}\right) - \int -\tfrac{1}{2}e^{-2y}dy$$
$$= (y+1)\left(-\tfrac{1}{2}e^{-2y}\right)\Big|_0^1 + \int_0^1 \tfrac{1}{2}e^{-2y}\,dy$$
$$= (y+1)\left(-\tfrac{1}{2}e^{-2y}\right) - \tfrac{1}{4}e^{-2y}\Big|_0^1$$
$$= 2\left(-\tfrac{1}{2}e^{-2}\right) - \tfrac{1}{4}e^{-2} - \left(1\left(-\tfrac{1}{2}\right) - \tfrac{1}{4}\right)$$

Ex $\int \sqrt[3]{t}\,\ln t\,dt$

<u>L</u>IATE

$u = \ln t \quad v = \frac{3}{4}t^{4/3}$

$du = \frac{1}{t}dt \quad dV = \sqrt[3]{t}\,dt = t^{1/3}dt$

$$= uv - \int v\,du$$
$$= \tfrac{3}{4}t^{4/3}\ln t - \int \tfrac{3}{4}t^{4/3}\cdot\tfrac{1}{t}dt$$
$$= \tfrac{3}{4}t^{4/3}\ln t - \int \tfrac{3}{4}t^{1/3}dt$$
$$= \boxed{\tfrac{3}{4}t^{4/3}\ln t - \tfrac{9}{16}t^{4/3} + C}$$