Test 1 Problem Possibilities

*solutions are in a separate pdf. I've compiled those mostly from tests online. There wasn't a single test that contained enough of the type of problems I expect from you.

Ex1 Determine the general solution $U(x,y)$ in terms of 2 arbitrary functions of the PDE given by

$$y U_{xy} + 2U_x = x$$

& then find a particular solution satisfying side condition:

$$U(x,1)=0 \quad \& \quad U(0,y)=0$$

Ex2 Solve the following PDE

$$U_x - 2U_t = 0 \text{ subject to } U(x,e^x)=$$
$$e^{2x}+4xe^x+4x^2$$

Ex3 Solve the PDE

$$x U_x - xt U_t = 0 \text{ for all } (x,y) \text{ if}$$
$$U(x,x) = x^2 e^{2x}$$

Ex 4 Is the function $u(x,y) = x^2 + y^2$
a solution to the PDE

$$y u_x - x u_y = 0?$$

Ex 5 Classify the PDE as
hyperbolic, elliptic, or parabolic

$$u_{xx} + 2u_{xt} + u_{tt} + u_x + u_t = 0$$

& find its general solution in terms
of 2 arbitrary functions

Ex 6 Classify the PDE as
hyperbolic, elliptic, or parabolic

$$2u_{xx} - 4u_{xy} - 6u_{yy} + u_x = 0$$

& then use transformations to write
it in its canonical form

Ex 7 Classify the PDE

$$2u_{xx} - 4u_{xy} - 6u_{yy} = 0$$ & find its general
solution in terms of 2 arbitrary functions

Ex 8 Solve the Poisson equation

$$u_{xx} + u_{yy} = x^2 + y^2, \quad x^2 + y^2 < 1$$

$$u(x,y) = 0, \quad x^2 + y^2 = 1$$

using polar coordinates. by looking for solutions of the form $u(x,y) = f(x^2 + y^2)$

Hint: $u_{xx} + u_{yy} = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta}$

Ex 9 Let $u(x,y)$ be the solution of the Dirichlet problem

$$u_{xx} + u_{yy} = 0 \quad \text{when} \quad x^2 + y^2 < 1$$

$$u(x,y) = 4x^3 \quad \text{when} \quad x^2 + y^2 = 1$$

Find the maximum value of $u(x,y)$ in the disk of radius 1 $(x^2 + y^2 \leq 1)$

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Ex 10 Let $u(r, \theta)$ be a solution

to $U_{rr} + \frac{1}{r} U_r + \frac{1}{r^2} U_{\theta\theta} = 0$ when $r^2 < 1$

$U(1, \theta) = 2 + 3\sin\theta$ when $r = 1$
$0 \leq \theta \leq 2\pi$

a) Find the minimum & maximum of u

on $x^2 + y^2 \leq 1$

b) Find the value of u at the origin

c) Does there exist a point in $x^2 + y^2 < 1$

that $u = 5$?

Note: these problems are just to give you idea of possible problem types. These don't include every possibility. Look at the review sheet & your webwork assignments for other types of problems