MA 241 - 050 Test 1 Version 1 Show all of your work

You may need the following on your test :

density of water = $1000 kg/m^3$
weight density of water = $62.4 lb/ft^3$
gravity = $9.8 m/s^2$
gravity = $32 ft/s^2$

1. (14 points) Find the length of the curve given by $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ from $x = 0$ to $x = 2$

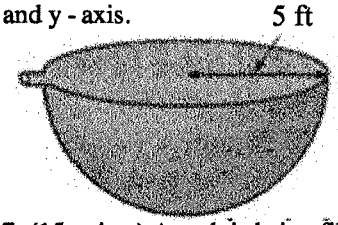
2. (14 points) A spring has a natural length of 10 inches. A 2 lb force stretches the spring to13 in. Find the work done in stretching the spring from 10 in. to 16 in. Include units with your answer.

3. (12 points) Find all values b so that the average value of $f(x) = 3x^2 - 8x + 7$ from $0 \leq x \leq b$ is 4

4. (15 points) A 100 ft rope hangs over the edge of a building. The rope weighs 20 lb. Find the work needed to pull half the rope to the top of thebuilding. Your answer should include units and a picture indicating the locations of your x - axis and y - axis.

5. (16 points) Find the centroid of the lamina bounded by $y = e^x$, the x - axis, the y - axis, and $x = 2$

6. (14 points) The hemispherical tank shown below is filled to a height of 3ft with water. Set up (**Do not evaluate**) the integral needed to find the work to pump all of the water out of the top of the tank. Your answer should include units and a picture indicating the locations of your x - axis and y - axis.



7. (15 points) A tank is being filled with water. The tank is sturdy except for triangular windows located at either end of the tank (one of these windows is pictured below). These windows can only withstand a maximum force of 42 N. Set up (**Do not evaluate**) the integral and equation needed to determine how high the water can reach before the triangular sides shatter. Your answer should include units and a picture indicating the locations of your x - axis and y - axis.

C2 T2 V[redacted] Solutions

1. (14 points)

$$y' = \frac{1}{2}(x^2+2)^{1/2}\, 2x$$

$$L = \int_0^2 \sqrt{1 + x^2(x^2+2)}\, dx$$

$$= \int_0^2 \sqrt{1 + x^4 + 2x^2}\, dx$$

$$= \int_0^2 \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^2 x^2+1\, dx = \frac{1}{3}x^3 + x \Big|_0^2$$

$$= \boxed{\frac{8}{3} + 2}$$

2. (14 points)

$$F = kx$$

$$2 = k\left(\frac{1}{4}\right) \quad k = 8$$

$$W = \int_0^{1/2} 8x\, dx = 4x^2 \Big|_0^{1/2} = 1 \text{ ft-lb}$$

3. (12 pts)

$$f_{ave} = \frac{\int_0^b 3x^2 - 8x + 7\,dx}{b} = 4$$

$$= \frac{x^3 - 4x^2 + 7x \Big|_0^b}{b} = 4$$

$$= b^2 - 4b + 7 = 4$$

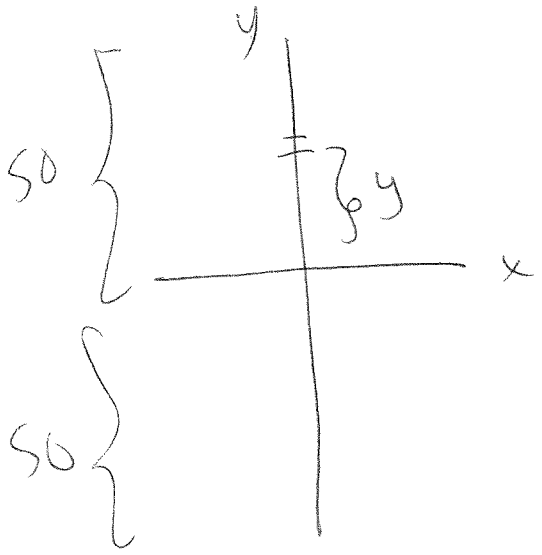
$$b^2 - 4b + 3 = 0$$

$$(b-3)(b-1) = 0$$

$$\boxed{b = 1,\ b = 3}$$

4. ~~(15 points)~~

(15 points)



$$W_{top} = \int_0^{50} \frac{1}{5}(50 - y)\,dy$$

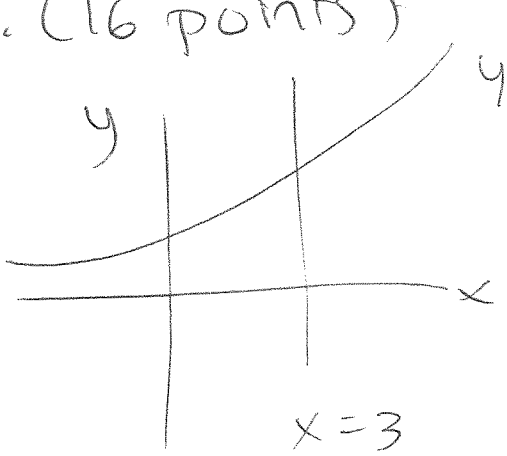
$$= \frac{1}{5}\left(50y - \frac{1}{2}y^2\right)_0^{50}$$

$$= \frac{1}{5}\left(\frac{50^2}{2}\right)$$

$$W_{bottom} = Fd = \frac{20}{100}(50)(50) = 500 \text{ ft-lb}$$

$$\boxed{\frac{1}{5}\frac{50^2}{2} + 500 \text{ ft-lb}}$$

5. (16 points)



$$\bar{x} = \frac{\int_0^3 x e^x \, dx}{\int_0^3 e^x \, dx}$$

LIATE $\quad u = x \quad v = e^x$

$du = dx \quad dv = e^x$

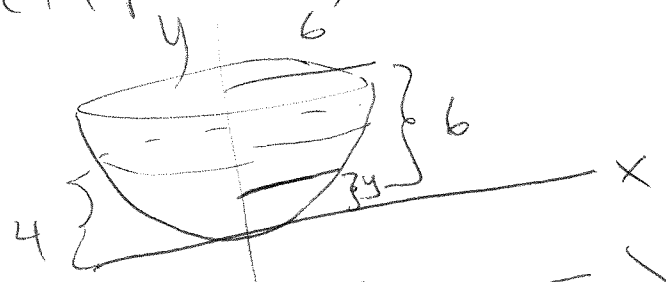
$$= \frac{x e^x \Big|_0^3 - \int_0^3 e^x \, dx}{e^x \Big|_0^3} = \frac{x e^x - e^x \Big|_0^3}{e^3 - 1} = \frac{2e^3 + 1}{e^3 - 1}$$

$$\bar{y} = \frac{\int_0^3 \frac{1}{2}\left[e^x\right]^2 dx}{e^3 - 1} = \frac{\int_0^3 \frac{1}{2} e^{2x} dx}{e^3 - 1} = \frac{\frac{1}{4} e^{2x} \Big|_0^3}{e^3 - 1}$$

$$= \frac{\frac{1}{4}\left(e^6 - 1\right)}{e^3 - 1}$$

$(\bar{x}, \bar{y})$

6. (14 points)

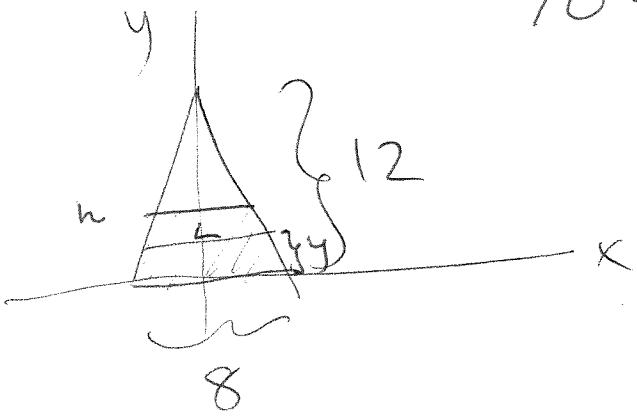


$(x)^2 + (y-6)^2 = 36$

$x = \sqrt{36-(y-6)^2}$

$$\int_0^4 62.4\left[\pi\left(\sqrt{36-(y-6)^2}\right)^2\right](6-y)\,dy \quad \text{ft-lb}$$

7. (15 points)



$$70 = \int_0^h 9.8(1000)\left[\frac{8}{12}(12-y)\right](h-y)\,dy$$

N

$$\frac{8}{12} = \frac{L}{12-y}$$