MA 242 Test 1 Version 1 No Work = No Credit! Put all answers in the bluebook.

1. (20 points) Use the points A(2,4,3), B(3,4,7), and the vector $\mathbf{c} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ to answer the following :
   a) Find the distance from B to the xz - plane
   b) Find the distance from B to the z - axis
   c) Find the area of the parallelogram with adjacent edges AB and $\mathbf{c} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

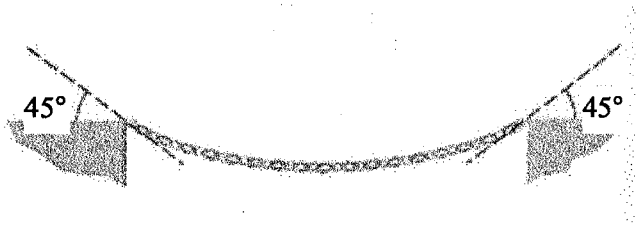
2. (15 points) Find a vector equation of the line of intersection of the planes $2x + 2y + 4z = 4$ and $x - 2y = -1$

3. (16 points) Use vectors $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{b} = -3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$ to answer the following :
   a) Find the angle between the vectors
   b) Find a unit vector in the opposite direction of vector **a**

4. (17 points) Find an equation of the plane that contains the line $x = 3 + 2t, y = 2 - t, z = 1 + t$ and is perpendicular to the plane $2x + 2y = 11$

5. (18 points) A ball leaves the ground at an angle of of 30° above the horizontal with an initial speed of 64 ft/s. Using the techniques we've discussed in class, find the following :
   a) Find the velocity vector **v**. Hint : $\vec{a} = \langle 0,0,-32 \rangle$
   b) Use your work from part a) to find the position vector **r**
   c) Find the location of the ball when it is at its maximum height

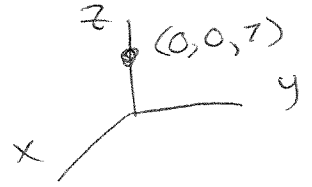
6. (14 points) The tension at each end of the chain has a magnitude 40 N. Find the tension vector $\vec{\mathbf{T}}_1$ on the left side of the chain and then find the mass of the chain. You may wish to use $g = 9.8 m/s^2$



C3 T1 V1 F23 Solutions

1. (20 points)

a) xz-plane $\rightarrow y = 0$ **4**

b) 

$\sqrt{(3-0)^2+(4-0)^2+(7-7)^2} = 5$

c)

$\overrightarrow{AB} = \langle 1, 0, 4 \rangle$

$$\overrightarrow{AB} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 4 \\ 2 & 3 & 0 \end{vmatrix}$$

$$= \langle -12, -(0-8), 3 \rangle$$

$$= \langle -12, 8, 3 \rangle$$

$$\sqrt{12^2+8^2+3^2}$$

2. (15 points)

$\vec{n_1} = \langle 2, 2, 4 \rangle$

$\vec{n_2} = \langle 1, -2, 0 \rangle$

$\vec{v} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 4 \\ 1 & -2 & 0 \end{vmatrix}$

$= \langle 0+8, -(0-4), -4-2 \rangle$

$= \langle 8, 4, -6 \rangle$

$z = 0 \quad 2x + 2y = 4$

$x - 2y = -1$

$3x = 3 \quad x = 1 \quad y = 1$

$\vec{r}(t) = \langle 1, 1, 0 \rangle + \langle 8, 4, -6 \rangle t$

3. (16 points)

a) $\vec{a} \cdot \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\theta$

$\langle 1,1,3 \rangle \cdot \langle -3,0,4 \rangle = \sqrt{1+1+9}\ 5 \cos\theta$

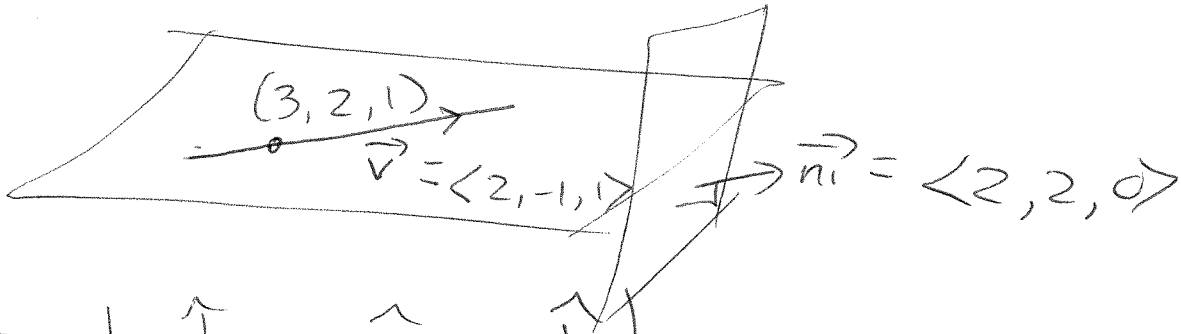
$-3 + 12 = \sqrt{11}\ 5 \cos\theta$

$$\theta = \cos^{-1}\left(\frac{9}{5\sqrt{11}}\right)$$

b)

$$\left\langle \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}} \right\rangle$$

4. (17 points)



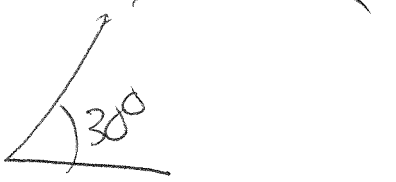
$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 0 \\ 2 & -1 & 1 \end{vmatrix}$$

$= \langle 2, -(2-0), -2-4 \rangle$

$= \langle 2, -2, -6 \rangle$

$$2(x-3) - 2(y-2) - 6(z-1) = 0$$

5. (18 points)

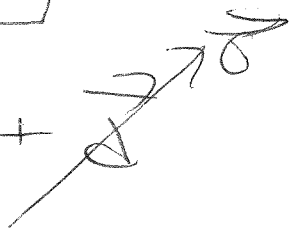


a) $\vec{v} = \langle 0, 0, -32t \rangle + \vec{c}$

$\vec{v}(0) = \langle 64\cos 30^\circ, 0, 64\sin 30^\circ \rangle$

$= \langle 64\sqrt{3}/2, 0, 64\,1/2 \rangle$

$\vec{v} = \langle 32\sqrt{3}, 0, 32 - 32t \rangle$

b) $\vec{r} = \langle 32\sqrt{3}t, 0, 32t - 16t^2 \rangle +$ 

c) $32 - 32t = 0 \rightarrow t = 1$

$\vec{r}(1) = \langle 32\sqrt{3}, 0, 32 - 16 \rangle$

6. (14 points) $\vec{T_1} = \langle -40\cos 45^\circ, 40\sin 45^\circ \rangle$

$= \langle -20\sqrt{2}, 20\sqrt{2} \rangle$

$\vec{T_1} + \vec{T_2} = \langle 0, \text{weight} \rangle$

$20\sqrt{2} + 20\sqrt{2} = \text{weight}$

$m = \dfrac{40\sqrt{2}}{9.8}$