Solutions

MA 341 Test 1 Version 1 No Work=No Credit!

1. (18 points) Consider the Initial Value Problem (IVP): $\frac{dy}{dx} = \frac{\cos^2 y}{2\sqrt{x+9}}$; $y(0) = 0$

Solve the IVP. Give your answer with y as an explicit function of x, if possible.

2. (19 points) Consider the IVP: $\frac{dy}{dx} = \frac{y - x}{x - 2x^2}$; $y(1) = 0$

a) Assuming $x > 0$, show $yx^{-1} + \ln(x) = 2y$ is an implicit solution to this IVP

b) Does the Existence and Uniqueness Theorem guarantee that this IVP has a unique solution?

3. (23 points) A mixing tank initially holds only 2 liters of pure water. At $t = 0$ a brine solution is pumped into the tank at a rate of 10 L/min. The solution in the tank is kept well-mixed and flows out of the tank at a rate of 8 L/min. If the concentration of salt entering the tank is 0.5 kg/L and $x(t)$ represents the amount of salt in the tank at time t, write the differential equation that describes this system and then solve it explicitly for $x(t)$.

4. (14 points) Use $\left(6x^2y^2 + \cos(2x)\right)dx + \left(bx^3y + 3e^{3y}\right)dy = 0$; $y(0) = 0$ to answer the following:

a) Find the value of b that will make this differential equation Exact

b) Find an implicit solution for the Initial Value Problem (IVP) for that value of b

5. (12 points) Solve the the IVP: $y'' + 6y' + 9y = 0$; $y(0) = 2, y'(0) = 5$

6. (14 points) Use $\frac{dy}{dt} = y^2(y - 6)$ to answer the following:

a) Is this differential equation separable, linear, both, or neither? (Just give a one word answer)

b) Sketch its phase line and classify all equilibria as we have done in class

c) Without solving the differential equation, find $y(t)$ if $y(0) = 6$

d) Use the phase line to predict the asymptotic behavior as $t \to \infty$ of the solution satisfying $y(0) = 4$

MA 341 T1 V1 Solutions

1. (18 points)

$$\frac{dy}{dx} = \frac{\cos^2 y}{2\sqrt{x+9}}$$

$$\int \frac{dy}{\cos^2 y} = \int \frac{1}{2}(x+9)^{-1/2}\,dx$$

$$\int \sec^2 y\,dy = \sqrt{x+9} + C$$

$$\tan y = \sqrt{x+9} + C$$

$$y = \tan^{-1}\left(\sqrt{x+9} + C\right)$$

$$y(0) = 0 = \tan^{-1}\left(\sqrt{9} + C\right)$$

$$C = -3$$

$$\boxed{y = \tan^{-1}\left(\sqrt{x+9} - 3\right)}$$

2. (19 points)

a) $yx^{-1} + \ln x = 2y$

$\frac{dy}{dx} x^{-1} - yx^{-2} + \frac{1}{x} = 2\frac{dy}{dx}$

$\frac{dy}{dx}\left(\frac{1}{x} - 2\right) = \frac{y}{x^2} - \frac{1}{x}$

$\frac{dy}{dx} = \frac{\frac{y}{x^2} - \frac{1}{x}}{\frac{1}{x} - 2} \cdot \frac{x^2}{x^2} = \frac{y-x}{x-2x^2}$ ✓

$\boxed{y(1)=0: \quad 0 \cdot 1^{-1} + \ln 1 = 2 \cdot 0}$ ✓

b) $f = \frac{y-x}{x-2x^2}$

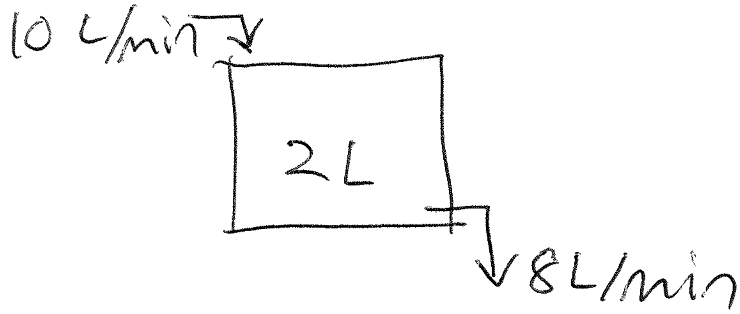
$\boxed{\text{cont at \& around } (1,0)}$

$\boxed{\frac{\partial f}{\partial y} = \frac{1}{x-2x^2}}$

$\boxed{\text{cont at \& around } (1,0)}$

Yes.

3. (23 points)



$$\frac{dx}{dt} = F_i C_i - F_o C_o$$

$$\boxed{\underbrace{\frac{dx}{dt} = 10(.5) - 8\left(\frac{x}{2+2t}\right)}_{2} \qquad \underbrace{x(0) = 0}}$$

$$= 5 - \frac{4x}{1+t}$$

17 pts for solving!

$$\frac{dx}{dt} + \frac{4x}{1+t} = 5$$

$$\mu(t) = e^{\int \frac{4}{1+t}\,dt} = e^{4\ln(1+t)} = (1+t)^4$$

$$\frac{d}{dt}\left[(1+t)^4 x\right] = 5(1+t)^4$$

$$(1+t)^4 x = \int 5(1+t)^4\,dt$$

$$= (1+t)^5 + C$$

$$x = \frac{(1+t)^5 + C}{(1+t)^4}$$

$x(0) = 0 \rightarrow C = -1$ $\quad \left.\boxed{x = \frac{(1+t)^5 - 1}{(1+t)^4}}\right\}2$

4. (14 points)

(4) a) $M_y = 12x^2y$ $N_x = 3bx^2y$

$\boxed{b = 4}$

(10) b) $F_x = 6x^2y^2 + \cos 2x$ $F_y = 4x^3y + 3e^{3y}$

$F = 2x^3y^2 + \frac{1}{2}\sin 2x + g(y)$

$F_y = 4x^3y + 0 + g'(y) \Rightarrow$

$g(y) = e^{3y}$

$$\boxed{2x^3y^2 + \frac{1}{2}\sin 2x + e^{3y} = C}$$

$y(0) = 0$

$0 + 0 + e^0 = C$

$$\boxed{2x^3y^2 + \frac{1}{2}\sin 2x + e^{3y} = 1}$$

5. (12 points)

$$r^2+6r+9=0$$

$$(r+3)^2=0$$

$$y=c_1e^{-3t}+c_2te^{-3t}$$

$$y(0)=2=c_1$$

$$y=2e^{-3t}+c_2te^{-3t}$$

$$y'=-6e^{-3t}+c_2e^{-3t}-3c_2te^{-3t}$$

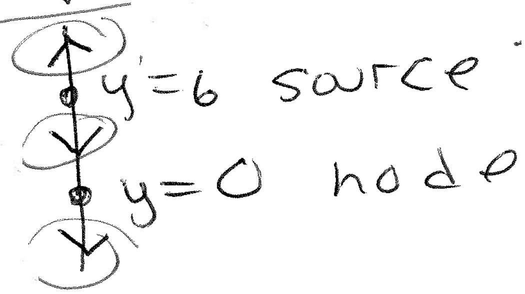
$$y'(0)=5=-6+c_2 \qquad c_2=11$$

$$\boxed{y=2e^{-3t}+11te^{-3t}}$$

6. (14 points)

a) separable

b)



c) $y(t)=6$

d) $y \to 0$