MA 241-050 Test 4: Be sure to show all of your work and **specify every test you use as well as the requirements for each test** as we have done in class.

1. (35 points) Determine if the following series converge or diverge. **Find the sum of convergent series.** Justify your answers thoroughly as we have done in class.

   a) $\sum_{n=1}^{\infty} \sqrt[n]{3}$

   b) $\sum_{n=1}^{\infty} tan^{-1}(n+1) - tan^{-1}(n)$ **Include the first three partial sums with your answer**

   c) $\sum_{n=0}^{\infty} \frac{12}{(-5)^n}$

2. (14 points) Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n[\ln(n)]^2}$ converges or diverges by the Integral Test. Briefly mention the two conditions we need to have before we can apply the Integral Test.

3. (22 points) Find the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{(x+1)^n}{2^n\sqrt{n+3}}$

4. (10 points) Determine if $\sum_{n=1}^{\infty} \frac{7+2(-1)^n}{n^3}$ converges or diverges

5. (14 points) Find a power series representation for f(x)=$\frac{x}{1+3x^2}$ and determine its interval of convergence. Fully simplify your series as we have done in class.

6. (5 points) If $\sum_{n=1}^{\infty} a_n = 7$, answer the following. **No explanations needed for this problem.**

   a) Let s_n be the nth partial sum of $\sum_{n=1}^{\infty} a_n$, what is $\lim_{n\to\infty} s_n$?

   b) What is $\lim_{n\to\infty} a_n$?

C2T4 Solutions

1. (35 points)

a) $\lim_{n\to\infty} 3^{1/n} = 3^0 = 1 \neq 0$ diverges Divergence test

b) $S_1 = \tan^{-1}2 - \tan^{-1}1$

$S_2 = \cancel{\tan^{-1}2} - \tan^{-1}1 + \tan^{-1}3 - \cancel{\tan^{-1}2}$

$S_3 = -\tan^{-1}1 + \cancel{\tan^{-1}3} + \tan^{-1}4 - \cancel{\tan^{-1}3}$

$S_n = -\tan^{-1}1 + \tan^{-1}(n+1)$

$\lim_{n\to\infty} S_n = -\tan^{-1}1 + \pi/2 = -\frac{\pi}{4} + \frac{\pi}{2} = \boxed{\frac{\pi}{4}}$ con[v] to

telescoping

c) $\sum_{n=0}^{\infty} \frac{12}{(-5)^n} = 12 - \frac{12}{5} + \frac{12}{(5)^2} - \frac{12}{5^3} + \ldots$

$a + ar + ar^2$

$a = 12 \quad r = -1/5$

Geometric $|-1/5| < 1$ conv to $\frac{a}{1-r}$

$= \frac{12}{1+\frac{1}{5}} = \frac{12}{\frac{6}{5}}$

$= \boxed{10}$

2. (14 points)

$\sum \frac{1}{n(\ln n)^2}$ positive, decreasing

$\int_2^\infty \frac{1}{x(\ln x)^2}\,dx \quad u=\ln x \quad du=\frac{1}{x}\,dx$

$\int_{\ln 2}^\infty \frac{1}{u^2}\,du = \lim_{t\to\infty} \int_{\ln 2}^t u^{-2}\,du$

$\lim_{t\to\infty} -\frac{1}{u}\Big|_{\ln 2}^t = \lim_{t\to\infty} -\frac{1}{t} + \frac{1}{\ln 2} = \frac{1}{\ln 2}$

$\sum \frac{1}{n(\ln n)^2}$ conv by Integral test.

3. (22 points)

Ratio test $\lim_{n\to\infty}\left|\dfrac{(x+1)^{n+1}}{2^{n+1}\sqrt{n+4}}\cdot\dfrac{2^n\sqrt{n+3}}{(x+1)^n}\right|$

$= \lim_{n\to\infty}\left|\dfrac{(x+1)\sqrt{n+3}}{2\sqrt{n+4}}\right| = \left|\dfrac{x+1}{2}\right| < 1 \quad |x+1| < 2$

$\boxed{R=2}$

$x = -1-2 = -3$

$x = -1+2 = 1$

$x=-3$: $\sum_{n=1}^{\infty}\dfrac{(-2)^n}{2^n\sqrt{n+3}} = \sum_{n=1}^{\infty}\dfrac{(-1)^n}{\sqrt{n+3}}$ AST

dec

$C_n = \dfrac{1}{\sqrt{n+3}} \geq C_{n+1} = \dfrac{1}{\sqrt{n+4}}$

$\lim_{n\to\infty}\dfrac{1}{\sqrt{n+3}} = 0$ ✓

CONV

$x=1$: $\sum_{n=1}^{\infty}\dfrac{2^n}{2^n\sqrt{n+3}} = \sum_{n=1}^{\infty}\dfrac{1}{\sqrt{n+3}}$

LCT $\lim_{n\to\infty}\dfrac{\frac{1}{\sqrt{n+3}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\dfrac{\sqrt{n}}{\sqrt{n+3}} = 1 > 0$ finite

$\sum\frac{1}{\sqrt{n}}$ p-series $p = 1/2 < 1$ div

$\sum\frac{1}{\sqrt{n+3}}$ div by LCT

Interval of C: $[-3, 1)$

4. (10 points)

$$\sum_{n=1}^{\infty} \frac{7+2(-1)^n}{n^3} \le \sum_{n=1}^{\infty} \frac{9}{n^3}$$ p-series $p=3>1$ conv

conv comparison test

5. (14 points)

$$x\left(\frac{1}{1-(-3x^2)}\right) = x\sum_{n=0}^{\infty}(-3x^2)^n$$

$$= x\sum_{n=0}^{\infty}(-3)^n x^{2n}$$

$$\boxed{=\sum_{n=0}^{\infty}(-3)^n x^{2n+1}}$$

$$|-3x^2| < 1$$

$$|x^2| < \frac{1}{3}$$

$$|x| < \frac{1}{\sqrt{3}}$$

Interval of C

$\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

6. (5 points)

a) 7

b) 0