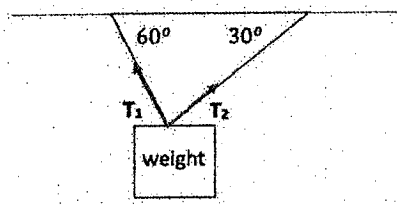
MA 242-050 Test 1 Version 1

1. (30 points) Use the points A(1,1,1), B(3, 2, 1), and C(5,1,2) to answer the following:
   a) Find a vector equation of the line segment AC
   b) Find the area of the triangle ABC
   c) Find the length of the side AB

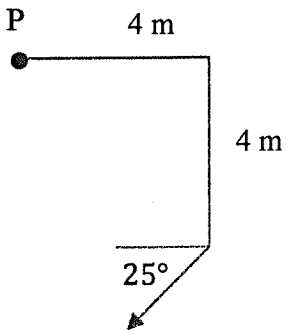
2. (28 points) Use the intersecting lines L_1: x=1+3t, y=2t, z=t-2 and L_2: x=10+s, y=6, z=1+2s to answer the following:
   a) Find an equation of the plane containing these lines
   b) Find the angle between the lines

3. (14 points) A ball is thrown at an angle of elevation of 30° above the horizontal with an initial speed v_0. The maximum height of the ball is 20 m. Use is $\vec{a} = \langle 0,-10 \rangle$ for the acceleration due to gravity.
   a) Find the velocity vector $\vec{v}$ (Your answer can have v_0 in it)
   b) Find the position vector $\vec{r}$ (Your answer can have v_0 in it)
   c) Find the initial speed v_0

4. (15 points) Use the picture below to answer the following:
   a) The magnitude of tension vector $\mathbf{T_1}$ is 12 lb. Write tension vector $\mathbf{T_1}$ in its component form
   b) Find the magnitude of tension vector $\mathbf{T_2}$
   c) Find the weight of the object suspended by these cables



5. (13 points) Find the magnitude of the torque about point P if a 24 N force is applied as shown



C3TIV1 Solutions

1. (30 points)

a) $\overrightarrow{AC} = \langle 5-1, 1-1, 2-1 \rangle = \langle 4, 0, 1 \rangle$

$$\vec{r}(t) = \langle 1,1,1 \rangle + \langle 4,0,1 \rangle t \qquad 0 \le t \le 1$$

b) $\overrightarrow{AB} = \langle 2, 1, 0 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{vmatrix}$$

$$= \langle 1, -(2-0), 0-4 \rangle$$

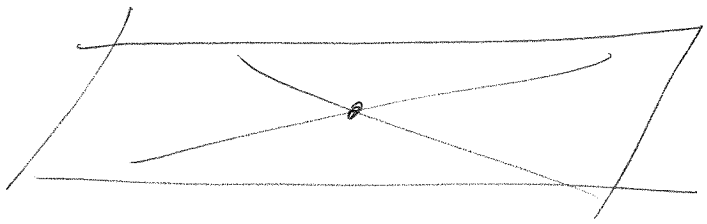
$$= \langle 1, -2, -4 \rangle$$

$$\sqrt{1^2+2^2+4^2}$$

$$\frac{\sqrt{21}}{2}$$

c) $\sqrt{5}$

2. (28 points)



$1+3t = 10+s \quad s=0$

$2t = 6 \rightarrow t=3$

$t-2 = 1+2s$

pt. $(10,6,1)$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix} = \langle 4, -(6-1), 0-2 \rangle$$

$$= \langle 4, -5, -2 \rangle$$

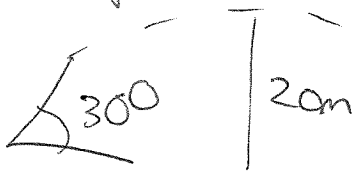
$$4(x-10) - 5(y-6) - 2(z-1) = 0$$

b) $\langle 3,2,1 \rangle \cdot \langle 1,0,2 \rangle = \sqrt{9+4+1}\ \sqrt{1+4}\cos\theta$

$3+2 = \sqrt{14}\ \sqrt{5}$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{14}\sqrt{5}}\right)$$

3. (14 points)



a) $\vec{v} = \langle 0, -10t \rangle + \vec{C}$

$\vec{v}(0) = \langle v_0 \cos 30^\circ, v_0 \sin 30^\circ \rangle$

$= \langle v_0 \sqrt{3}/2, v_0 \, 1/2 \rangle$

$\vec{v} = \langle v_0 \sqrt{3}/2, v_0/2 - 10t \rangle$

b) $\vec{r} = \langle v_0 \sqrt{3}/2 \, t, \frac{v_0}{2} t - 5t^2 \rangle$

c) $\frac{v_0}{2} - 10t = 0$

$\frac{v_0}{2} = 10t$

$\frac{v_0}{2} t - 5t^2 = 20$

$10t^2 - 5t^2 = 20$

$5t^2 = 20$

$t^2 = 4 \quad t = 2$

$\boxed{v_0 = 40}$

4. (15 points)

a) $\vec{T_1} = \langle -12\cos 60^\circ, 12\sin 60^\circ \rangle$

$= \langle -12 \cdot \frac{1}{2}, 12 \cdot \frac{\sqrt{3}}{2} \rangle$

$= \langle -6, 6\sqrt{3} \rangle$

b) $\vec{T_1} + \vec{T_2} = \langle 0, \text{weight} \rangle$

$\vec{T_2} = \langle T_2 \cos 30^\circ, T_2 \sin 30^\circ \rangle$

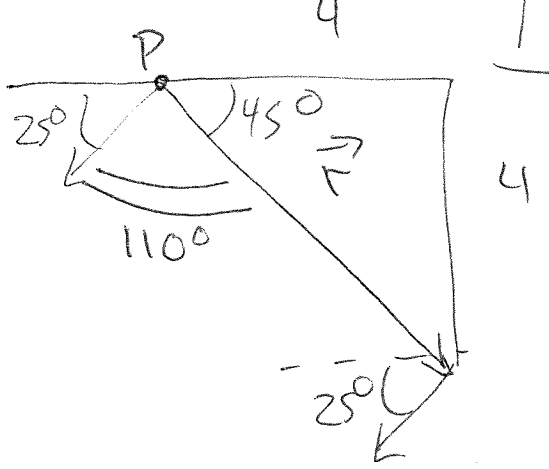
$= \langle T_2 \cdot \frac{\sqrt{3}}{2}, T_2 \cdot \frac{1}{2} \rangle$

$-6 + T_2 \cdot \frac{\sqrt{3}}{2} = 0$

$T_2 = \frac{12}{\sqrt{3}}$

c) $\text{weight} = 6\sqrt{3} + \frac{12}{\sqrt{3}}\left(\frac{1}{2}\right)$

5. (13 points)



$\|\vec{r}\| = 24\sqrt{32}\sin 110^\circ$