MA 341 Test 2 Version 1

1. (17 points) Use **the method of undetermined coefficients** to solve the Initial Value Problem (IVP): $y'' - 6y' + 10y = -10e^{2t}$; y(0)=0, $y'(0)$=0

2. (13 points)
   Use **the method of variation of parameters** to find the particular solution to:
   $y'' - 2y' + y = \frac{e^t}{t}$

3. (12 points) A 64 lb weight attached to a spring stretches it 6 inches before coming to a rest at equilibrium. The damping constant is 1 lb - sec/ft. At time t = 0, the spring is compressed 2 inches and released. If y(t) is the position of the mass at time t, use 32 ft/s^2 for the gravitational constant and formulate the IVP that describes this system

4. (12 points) Use the definition of the Laplace transform to find the Laplace transform of f(t)=7 and determine its domain

Use the table below to answer the following problems:

$L\{y''\} = s^2L\{y\} - sy(0) - y'(0)$	$L\{\cos(bt)\} = \frac{s}{s^2 + b^2}$	$L\{e^{at}\} = \frac{1}{s - a}$
$L\{y'\} = sL\{y\} - y(0)$	$L\{\sin(bt)\} = \frac{b}{s^2 + b^2}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$
$L\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$	$L\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2 + b^2}$	$L\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2 + b^2}$
$L\{g(t)u(t-a)\} = e^{-as}L\{g(t+a)\}$	$L^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	$L\{t^n f(t)\} = (-1)^n \frac{d^n(F(s))}{ds^n}$
	$L\{1\} = \frac{1}{s}$	$L\{u(t-a)\} = \frac{e^{-as}}{s}$

5. (7 points) Express the given function using unit step functions $f(t) = \begin{cases} 12 & t < 3 \\ 10t & 3 \le t \end{cases}$

6. (10 points) Find the inverse Laplace of the following: $\frac{8e^{-3s}}{(s-2)}$

7. (14 points) Find the inverse Laplace of the following: $\frac{10+5s}{s(s^2-2s+10)}$

8. (15 points) Use the method of Laplace transforms to solve the Initial Value Problem:
   $y' + 3y = 18t$; y(0)=0

341 T1 V1 Solutions

1. (17 points) $r^2 - 6r + 10 = 0$

$$r = \frac{6 \pm \sqrt{36-40}}{2} = 3 \pm i$$

$y_c = e^{3t}\left[c_1 \cos t + c_2 \sin t\right]$

$y_p = Ae^{2t} \quad y_p' = 2Ae^{2t} \quad y_p'' = 4Ae^{2t}$

$y'' - 6y' + 10y = -10e^{2t}$

$4Ae^{2t} - 6(2Ae^{2t}) + 10Ae^{2t} = -10e^{2t}$

$4A - 12A + 10A = -10$

$A = -5$

$y = e^{3t}\left[c_1 \cos t + c_2 \sin t\right] - 5e^{2t}$

$y(0) = 0 = c_1 - 5 \quad c_1 = 5$

$y = e^{3t}\left[5\cos t + c_2 \sin t\right] - 5e^{2t}$

$y' = 3e^{3t}\left[5\cos t + c_2 \sin t\right] + e^{3t}\left[-5\sin t + c_2 \cos t\right]$
$- 10e^{2t}$

$y'(0) = 0 = 3 \cdot 5 + c_2 - 10$

$c_2 = -5$

$$\boxed{y = e^{3t}\left[5\cos t - 5\sin t\right] - 5e^{2t}}$$

2. (13 points) $r^2 - 2r + 1 = 0$

$$(r-1)^2 = 0$$

$$y_c = c_1 e^t + c_2 t e^t$$

$$-\left(v_1' e^t + v_2' t e^t = 0 \right)$$

$$v_1' e^t + v_2' \left(1e^t + te^t \right) = \frac{e^t}{t}$$

$$v_2' e^t = \frac{e^t}{t}$$

$$v_2' = \frac{1}{t} \qquad v_2 = \ln|t|$$

$$v_1' = -v_2' t$$

$$v_1' = -\frac{1}{t} t = -1 \qquad v_1 = -t$$

$$\boxed{y_p = -t e^t + \ln|t| \, t e^t}$$

3. (12 points)

$W = mg$

$64 = m\,32 \quad m = 2$

$F = ky$

$64 = k\left(\frac{1}{2}\right) \quad k = 128$

$$\boxed{2y'' + 1y' + 128y = 0 \quad y(0) = -\frac{1}{6} \quad y'(0) = 0}$$

4. (12 points)

$$\mathcal{L}\{7\} = \int_0^\infty e^{-st}\,7\,dt = \lim_{n\to\infty} \int_0^n e^{-st}\,7\,dt$$

$$= \lim_{n\to\infty} \frac{-7}{s} e^{-st}\Big|_0^n = \lim_{n\to\infty} \frac{-7}{s}\left[e^{-sn} - e^0\right]$$

$$= \boxed{\frac{7}{s}, \quad s > 0}$$

5. (~~10~~ 7 points)

$$f = 12 + (10t - 12)u(t-3)$$

6. (10 points) $F(s) = \dfrac{8}{s-2}$ $f(t) = 8e^{2t}$

$$f(t-3)u(t-3)$$

$$\boxed{8e^{2(t-3)}u(t-3)}$$

7. (1~~4~~ points) $\dfrac{10+5s}{s(s^2-2s+10)}$

$$(s-\alpha)^2 + \beta^2$$

$$s^2 - 2\alpha s + \alpha^2 + \beta^2$$

$$\alpha = 1, \quad \beta = 3$$

$$\frac{A}{s} + \frac{B(s-1) + C3}{(s-1)^2 + 9}$$

$$As^2 - 2As + 10A + Bs^2 - Bs + 3Cs = 10 + 5s$$

$A + B = 0$

$-2A - B + 3C = 5$ $\quad -2\cdot 1 + 1 + 3C = 5$

$10A = 10$ $\quad A = 1,\ B = -1,\ C = 2$

$$\boxed{1 - e^t\cos 3t + 2e^t\sin 3t}$$

8. (15 points)

$$sL - \cancel{y(0)} + 3L = \frac{18}{s^2}$$

$$(s+3)L = \frac{18}{s^2}$$

$$L = \frac{18}{s^2(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3}$$

$$As(s+3) + B(s+3) + Cs^2 = 18$$

$$As^2 + \underline{3As} + \underline{Bs} + 3B + Cs^2 = 18$$

$$A + C = 0$$

$$3A + B = 0$$

$$3B = 18$$

$$B = 6 \quad A = -2 \quad C = 2$$

$$L = \frac{-2}{s} + \frac{6}{s^2} + \frac{2}{s+3}$$

$$\boxed{y = -2 + 6t + 2e^{-3t}}$$