MA 341 Test 2 Version 2

1. (17 points) Use **the method of undetermined coefficients** to solve the Initial Value Problem (IVP): $y'' - 6y' + 10y = 6e^{2t}$; y(0)=0, $y'(0)$=0

2. (13 points) Use **the method of variation of parameters** to find a particular solution to: $y'' - 2y' + y = \frac{e^t}{t}$

3. (12 points) A 64 lb weight attached to a spring stretches it 6 inches before coming to a rest at equilibrium. The damping constant is 3 lb - sec/ft. At time t = 0, the spring is compressed 3 inches and released. If y(t) is the position of the mass at time t, use 32 ft/s^2 for the gravitational constant and formulate the IVP that describes this system. **(Do not solve it)**

4. (12 points) Use the definition of the Laplace transform to find the Laplace transform of f(t)=11 and determine its domain

Use the table below to answer the following problems:

$L\{y''\} = s^2L\{y\} - sy(0) - y'(0)$	$L\{\cos(bt)\} = \frac{s}{s^2+b^2}$	$L\{e^{at}\} = \frac{1}{s-a}$
$L\{y'\} = sL\{y\} - y(0)$	$L\{\sin(bt)\} = \frac{b}{s^2+b^2}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$
$L\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$	$L\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2+b^2}$	$L\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2+b^2}$
$L\{g(t)u(t-a)\} = e^{-as}L\{g(t+a)\}$	$L^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	$L\{t^n f(t)\} = (-1)^n \frac{d^n(F(s))}{ds^n}$
	$L\{1\} = \frac{1}{s}$	$L\{u(t-a)\} = \frac{e^{-as}}{s}$

5. (7 points) Express the given function using unit step functions $f(t) = \begin{cases} 8 & t < 7 \\ 4t & 7 \le t \end{cases}$

6. (10 points) Find the inverse Laplace of the following: $\frac{6e^{-4s}}{(s-3)}$

7. (14 points) Find the inverse Laplace of the following: $\frac{30+6s}{s(s^2-2s+10)}$

8. (15 points) Use the method of Laplace transforms to solve the Initial Value Problem: $y' + 2y = 12t$; y(0)=0

MA 341 T2 V2 Solutions

1. (17 points) $r^2-6r+10=0$

$$r=\frac{6\pm\sqrt{36-40}}{2}=\frac{6\pm 2i}{2}=3\pm i$$

$$y_c=e^{3t}\left[c_1\cos t+c_2\sin t\right]$$

$$y_p=Ae^{2t} \quad y_p'=2Ae^{2t} \quad y_p''=4Ae^{2t}$$

$$y''-6y'+10y=6e^{2t}$$

$$4Ae^{2t}-12Ae^{2t}+10Ae^{2t}=6e^{2t}$$

$$2A=6 \quad A=3$$

$$y=e^{3t}\left[c_1\cos t+c_2\sin t\right]+3e^{2t}$$

$$y(0)=0=c_1+3 \quad c_1=-3$$

$$y=e^{3t}\left[-3\cos t+c_2\sin t\right]+3e^{2t}$$

$$y'=3e^{3t}\left[-3\cos t+c_2\sin t\right]+e^{3t}\left[3\sin t+c_2\cos t\right]+6e^{2t}$$

$$y'(0)=0=-9+c_2+6 \quad c_2=3$$

$$\boxed{y=e^{3t}\left[-3\cos t+3\sin t\right]+3e^{2t}}$$

2. (13 points)

$$r^2-2r+1=0$$
$$(r-1)^2=0$$
$$y_c = c_1e^t + c_2te^t$$
$$v_1'e^t + v_2'te^t = 0$$
$$v_1'e^t + v_2'(e^t+te^t) = \frac{e^t}{t}$$

$$v_2'e^t = \frac{e^t}{t} \qquad v_2' = \frac{1}{t} \qquad v_2 = \ln|t|$$

$$v_1' = -v_2't$$
$$= -\frac{1}{t}\cdot t = -1$$
$$v_1 = -t$$

$$\boxed{y_p = -te^t + \ln|t|\,te^t}$$

3. (12 points)

$W = mg$ $\qquad F = ky$

$64 = m(32)$ $\quad m = 2$ $\qquad 64 = k(1/2)$ $\qquad k = 128$

$b = 3$

$$2y'' + 3y' + 128y = 0 \quad y(0) = -\frac{1}{4} \quad y'(0) = 0$$

4. (12 points)

$$\mathcal{L}\{11\} = \int_0^\infty e^{-st}\, 11\, dt$$

$$= \lim_{n\to\infty} \int_0^n e^{-st}\, 11\, dt$$

$$= \lim_{n\to\infty} -\frac{11}{s} e^{-st}\Big|_0^n = \lim_{n\to\infty} -\frac{11}{s}\left[e^{-sn} - \cancel{1}\right]$$

$$= -\frac{11}{s}(-1) = \boxed{\frac{11}{s}}$$

$\boxed{s > 0}$ domain

5. (7 points) $f = 8 + (4t-8)u(t-7)$

6. (10 points) $F(s) = \dfrac{6}{s-3}$ $\quad f(t) = 6e^{3t}$

$$\mathcal{L}^{-1}\left\{\frac{6e^{-4s}}{s-3}\right\} = f(t-4)u(t-4)$$

$$= \boxed{6e^{3(t-4)}u(t-4)}$$

7. (14 points) $s^2-2s+10 = (s-\alpha)^2+\beta^2$

$\alpha = 1 \quad \beta = 3$

$$\frac{A}{s} + \frac{B(s-1)+C\cdot 3}{(s-1)^2+3^2}$$

$$As^2 - \underline{2As} + 10A + (B(s-1)+C\cdot 3)s = 30 + 6s$$

$$Bs^2 - \underline{Bs} + \underline{3Cs}$$

$A+B=0$

$-2A - B + 3C = 6 \qquad -6+3+3C = 6$

$10A = 30 \quad A = 3 \quad B = -3 \qquad C = 3$

$$\boxed{3 - 3e^t\cos 3t + 3e^t \sin 3t}$$

8. (15 poins)

$$sL - \cancel{y(0)} + 2L = \frac{12}{s^2}$$

$$L = \frac{12}{s^2(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

$$As(s+2) + B(s+2) + Cs^2 = 12$$

$$As^2 + Cs^2 = 0$$

$$2As + Bs = 0$$

$2B = 12 \quad B = 6 \quad A = -3 \quad C = 3$

$$L = \frac{-3}{s} + \frac{6}{s^2} + \frac{3}{s+2}$$

$$\boxed{y = -3 + 6t + 3e^{-2t}}$$