MA 242-050 Test 3 Version 1

1. (13 points) Evaluate $\int_0^2 \int_1^{5-x^2} \frac{4}{\sqrt{5-y}}\, dydx$ by switching the order of integration

2. (15 points) **Set up** the integral needed to find the mass of the solid bounded by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$ with density $\sigma(x, y, z) = z^2$ using augmented cylindrical coordinates. **Do not evaluate.**

3. (14 points) **Set up** the integrals needed to find the average value of f(x,y)= 10x+y over the lamina bounded by the triangular region D with vertices (1,1), (4,1), (4,7). **Do not evaluate.**

4. (29 points) Use polar coordinates for the following problems:
   a) **Set up** the integral required to find the mass of the lamina bounded by x=2 and $x^2 + y^2 = 9$, where x $\geq$ 2 if the density of the lamina is 3 times its distance to the origin. **Do not evaluate.**

   b) Evaluate $\iint_D \frac{x}{(x^2+y^2)^2}\, dA$ where D is the region in the xy-plane where $x^2 + y^2 \geq 1$ (Please note that this is *greater* than or equal to 1), with x $\geq$ 0, y $\geq$ 0

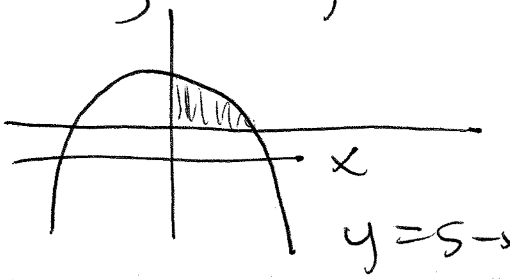
5. (17 points) Evaluate $\iiint_F z^3\sqrt{x^2 + y^2 + z^2}\,dV$ where F is the solid hemisphere bounded by z=$\sqrt{4 - x^2 - y^2}$ and the xy-plane

6. (12 points) Find the volume of the solid bounded by the cylinder z=$17 - x^2$, the planes z=1 and y=6 in the first octant.

MA 242-050 Test 3 V1 Solutions

1. (13 points)

$1 \le y \le 5 - x^2$



$x = \sqrt{5-y}$

$$\int_1^5 \int_0^{\sqrt{5-y}} \frac{4}{\sqrt{5-y}}\,dx\,dy$$

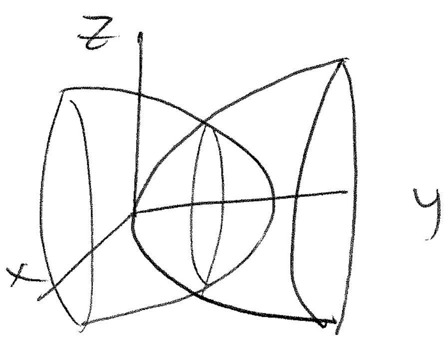
$$= \int_1^5 \left.\frac{4x}{\sqrt{5-y}}\right|_0^{\sqrt{5-y}} dy$$

$$= \int_1^5 4\,dy = 4y\Big|_1^5$$

$$= \boxed{16}$$

2. (15 points)

$$m = \iiint_E z^2\,dV = \int_0^{2\pi}\int_0^2\int_{r^2}^{8-r^2} r^2\sin^2\theta\,dy\,r\,dr\,d\theta$$

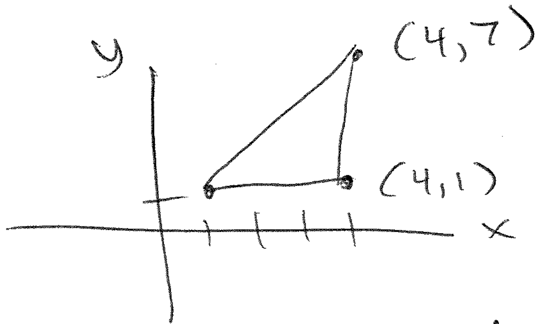


$8 - r^2 = r^2$

$8 = 2r^2$

$4 = r^2$

3. (14 points)



$$m = \frac{\Delta y}{\Delta x} = \frac{7-1}{4-1} = \frac{6}{3} = 2$$

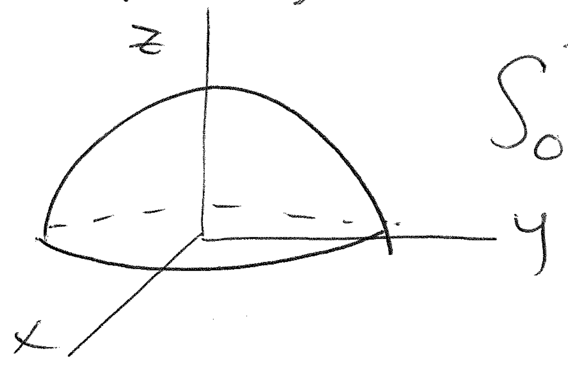
$$y = mx + b \qquad y = 2x + b$$

$$1 = 2\cdot 1 + b \qquad b = -1$$

$$y = 2x - 1$$

$$\boxed{\frac{\int_1^4 \int_1^{2x-1} 10x + y \, dy \, dx}{\int_1^4 \int_1^{2x-1} 1 \, dy \, dx}}$$

5. (17 points)



$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{2} \rho^3\cos^3\phi\sqrt{\rho^2}\;\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$2\pi\int_0^{\pi/2}\cos^3\phi\sin\phi\, d\phi \int_0^2 \rho^6\, d\rho$$

$u=\cos\phi$

$du=-\sin\phi$

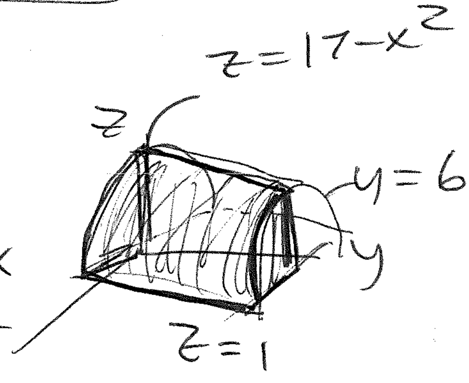
$$2\pi\int_1^0 -u^3\, du \;\; \frac{1}{7}\rho^7\Big|_0^2$$

$$2\pi\int_0^1 u^3\, du \;\; \frac{1}{7}\cdot 2^7$$

$$\boxed{2\pi\,\frac{1}{4}\cdot\frac{1}{7}\cdot 2^7}$$

6. (12 points)

$$V=\int_0^4\int_0^6 17-x^2-1\, dy\, dx$$



$17-x^2=1$

$16=x^2$

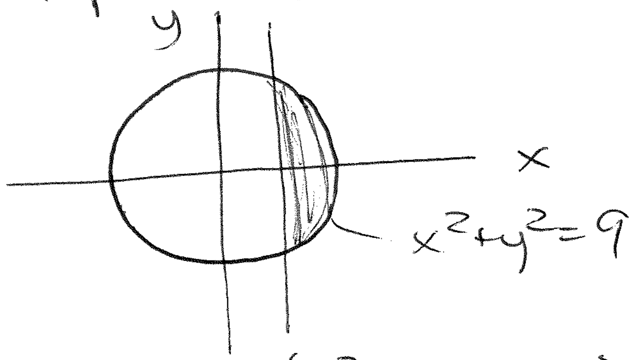
$$\int_0^4 16-x^2\, dx\int_0^6 dy$$

$$16x-\frac{1}{3}x^3\Big|_0^4\cdot 6$$

$$\boxed{\left(64-\frac{64}{3}\right)\cdot 6}$$

4. (29 points)

a)



$\longrightarrow$ $r\cos\theta = 2$

$r = 2/\cos\theta$

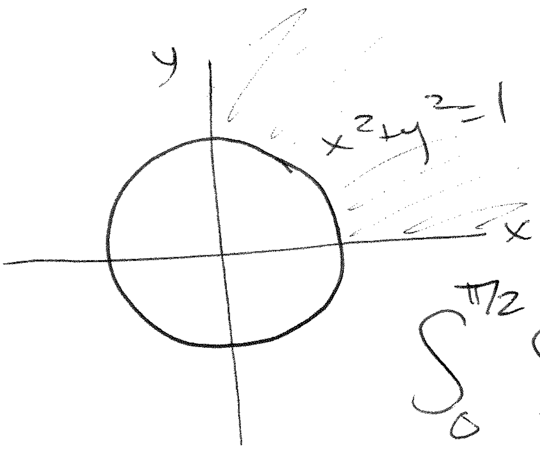
$$m = \iint 3\sqrt{x^2+y^2}\, dA$$

$$\int_{-\cos^{-1}(\frac{2}{3})}^{+\cos^{-1}(\frac{2}{3})} \int_{\frac{2}{\cos\theta}}^{3} 3\, r\, r\, dr\, d\theta$$

$3 = \frac{2}{\cos\theta}$

$\cos\theta = 2/3 \qquad \theta = \cos^{-1}(\frac{2}{3})$

b)



$$\int_0^{\pi/2} \int_1^{\infty} \frac{r\cos\theta}{(r^2)^2}\, r\, dr\, d\theta$$

$$\int_0^{\pi/2} \cos\theta\, d\theta \int_1^{\infty} \frac{1}{r^2}\, dr$$

$$\sin\theta \Big|_0^{\pi/2} \qquad \lim_{n\to\infty} \int_1^{n} \frac{1}{r^2}\, dr$$

$$1 \qquad \lim_{n\to\infty} -\frac{1}{r} \Big|_1^{n} = \lim_{n\to\infty} -\frac{1}{n} + 1 = \boxed{1}$$