MA 241-050 Test 1 Version 1

You may need the following on your test:

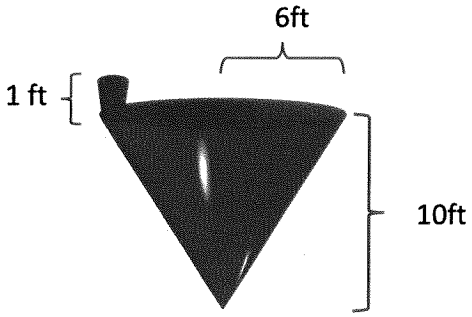
density of water $= 1000\ kg/m^3$
weight density of water $= 62.4\ lb/ft^3$
gravity $= 9.8\ m/s^2$
gravity $= 32\ ft/s^2$

1. (14 points) Find the average value of f(x)= $9x^2 \ln(x)$ from $1 \leq x \leq 3$

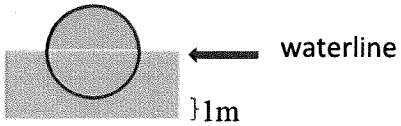
2. (14 points) Assuming 30 J of work is needed to stretch a spring from its natural length of 400 cm to a length of 500 cm, answer the following:
   a) Find the work needed to stretch the spring from 500 cm to 600 cm. Include units with your answer.
   b) How much force is required to keep the spring stretched to 600 cm. Include units with your answer.

3. (14 points) A lamina in the first quadrant is bounded by y=6x, the x-axis, and x=b.
   a) Find its centroid. Show your work.
   b) Using your answer from part a), find its centroid if b=3.

4. (15 points) A tank has the shape of an inverted circular cone with a height of 10 ft and a base radius of 6 ft. It is filled with water to a height of 8 ft. Set up the integral needed to find the work required to pump all the water out of a spout located 1 ft spout above the tank. Your answer should include units and a picture with locations of the x and y axes.



5. (15 points) Set up the integral needed to find the hydrostatic force on the submerged semicircular region with radius 3m pictured below. Your answer should include units and a picture with locations of the x and y axes.



6. (14 points) A 1600 lb elevator is suspended by a 100 ft cable that weights 1000 lbs. How much work is required to raise the elevator 100 ft? Include units with your answer.

7. (14 points) Find the length of the curve given by $x = e^t - t,\ y = 4e^{(t/2)},\ 0 \leq t \leq 2$

C2 T1 V1 Solutions

1. (14 points)

$$f_{ave} = \frac{\int_1^3 9x^2 \ln x \, dx}{3-1}$$

$$= \frac{1}{2}\int_1^3 9x^2 \ln x \, dx$$

LIATE

$u = \ln x$ $\quad v = 3x^3$

$du = \frac{1}{x}dx$ $\quad dV = 9x^2 dx$

$uv - \int v\,du$

$$\frac{1}{2}\left[3x^3 \ln x - \int 3x^2 dx\right]$$

$$\frac{1}{2}\left[3x^3 \ln x - x^3\right]_1^3 = \frac{1}{2}\left[81\ln 3 - 27 - [0-1]\right]$$

2. (14 points)

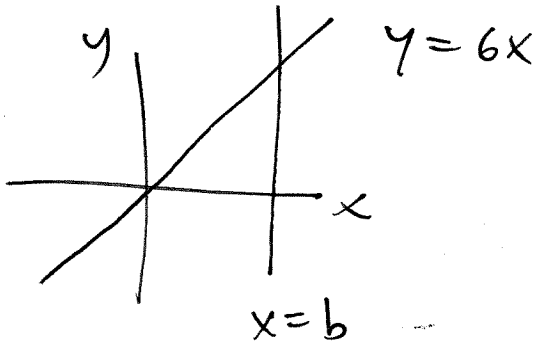
a) $W = \int_a^b kx\,dx$ $\quad 30 = \int_0^{1m} kx\,dx$

$30 = \frac{1}{2}kx^2\Big|_0^1 = \frac{1}{2}k$ $\quad k = 60$

$W = \int_1^2 60x\,dx = 30x^2\Big|_1^2 = 30[4-1] = \boxed{90\ J}$

b) $F = 60(2) = 120\ N$

3. (14 points)



a)

$$\bar{x} = \frac{\int_0^b x(6x)\,dx}{\int_0^b 6x\,dx} = \frac{2x^3\Big|_0^b}{3x^2\Big|_0^b} = \frac{2b^3}{3b^2} = \frac{2}{3}b$$

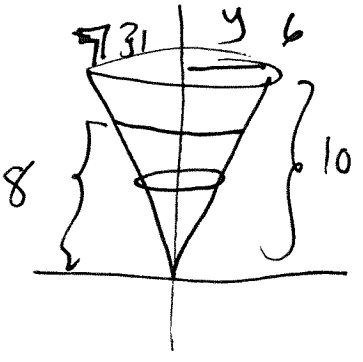
$$\bar{y} = \frac{\int_0^b \frac{1}{2}[6x]^2\,dx}{3b^2} = \frac{\int_0^b 18x^2\,dx}{3b^2} = \frac{6x^3\Big|_0^b}{3b^2}$$

$$= 2b$$

$$\left(\tfrac{2}{3}b, 2b\right)$$

b) $(2, 6)$

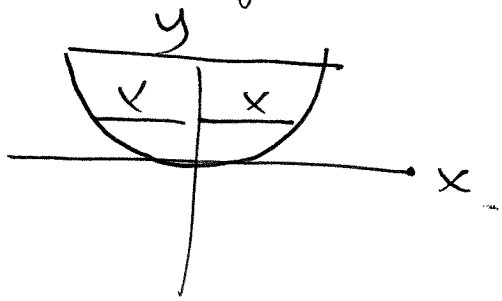
4.



$$W = \int_0^8 62.4\left[\pi\left(\tfrac{6}{10}y\right)^2\right](11-y)\,dy \quad \text{ft-lb}$$

$$\frac{r}{y} = \frac{6}{10} \quad \Rightarrow \quad r = \frac{6}{10}y$$

5. (15 points)



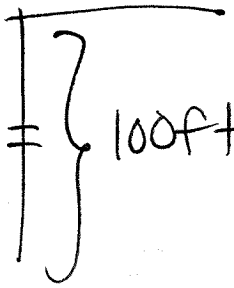
$$(x-0)^2+(y-3)^2=9$$

$$x=\sqrt{9-(y-3)^2}$$

$$F=\int_0^3 1000(9.8)\left[2\sqrt{9-(y-3)^2}\right](3-y)\,dy \text{ N}$$

6. (14 points)

$W_{elevator}=1600(100)=160{,}000$ ft-lb



$$W_{cable}=\int_0^{100} 10\,(100-y)\,dy$$

$$=10\left(100y-\tfrac{1}{2}y^2\right)_0^{100}$$

$$10\left(\frac{100^2}{2}\right) \text{ ft-lb}$$

$$\frac{1000\text{lb}}{100\text{ft}}=\frac{10\text{lb}}{\text{ft}}$$

$$160{,}000+10\left(\frac{100^2}{2}\right) \text{ ft-lb}$$

7. (14 points)

$\frac{dx}{dt} = e^t - 1 \qquad \frac{dy}{dt} = 4 \cdot \frac{1}{2} e^{t/2} = 2e^{t/2}$

$$L = \int_0^2 \sqrt{(e^t-1)^2 + (2e^{t/2})^2}\, dt$$

$$= \int_0^2 \sqrt{e^{2t} - 2e^t + 1 + 4e^t}\, dt$$

$$\int_0^2 \sqrt{e^{2t} + 2e^t + 1}\, dt$$

$$= \int_0^2 \sqrt{(e^t+1)^2}\, dt$$

$$= \int_0^2 e^t + 1\, dt = e^t + t \Big|_0^2$$

$$e^2 + 2 - \cancel{e^0}^{1} = \boxed{e^2 + 1}$$