MA 242 Test 1 Version 1

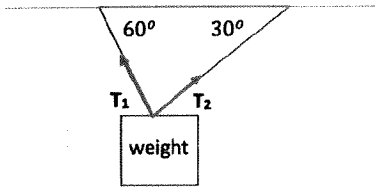
1. (20 points) Use vectors $\vec{a} = \langle 1, -2, 1 \rangle$ and $\vec{b} = \langle 2, 3, 1 \rangle$ to answer the following:
   a) Find a vector in the same direction as $\vec{a}$, but with magnitude 7
   b) Find the area of the parallelogram with adjacent edges $\vec{a}$ and $\vec{b}$
   c) Find the angle between $\vec{a}$ and $\vec{b}$

2. (20 points) Find an equation of the plane containing the point A(2,3,4) and the line x=1+t, y=4-t, z= 7+2t

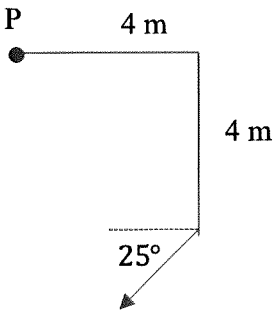
3. (12 points) Find parametric equations of the line through (2,8, -6) and perpendicular to the plane 11x-4y+8z=10

4. (20 points) A ball is thrown from the ground at an angle of elevation of 45° above the horizontal with an initial speed v_0. The ball lands 20 m away. Use is $\vec{\mathbf{a}} = \langle 0, -10 \rangle$ for the acceleration due to gravity.
   a) Find the velocity vector $\vec{\mathbf{v}}$ (Your answer can have v_0 in it)
   b) Find the position vector $\vec{\mathbf{r}}$ (Your answer can have v_0 in it)
   c) Find the initial speed v_0

5. (15 points) A 16 lb weight is suspended from two cables as shown below.
   a) Write tension vector $\mathbf{T_1}$ in its component form.
   b) Find the magnitude of the tension in each cable



6. (13 points) Find the magnitude of the torque about point P if a 24 N force is applied as shown



C3 T1 V1 Solutions

1. (20 points)

a) $\|\vec{a}\| = \sqrt{1+4+1} = \sqrt{6}$

$$7\hat{a} = 7\frac{\langle 1,-2,1\rangle}{\sqrt{6}} = \boxed{\left\langle \frac{7}{\sqrt{6}}, \frac{-14}{\sqrt{6}}, \frac{7}{\sqrt{6}} \right\rangle}$$

b) $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = \langle -2-3, -(1-2), 3+4\rangle$

$= \langle -5, 1, 7\rangle$

$\|\vec{a}\times\vec{b}\| = \sqrt{25+1+49} = \boxed{\sqrt{75}}$

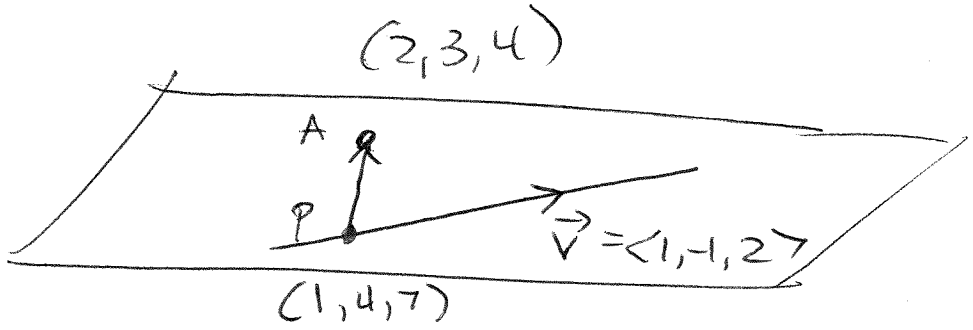
c) $\vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta$

$\langle 1,-2,1\rangle \cdot \langle 2,3,1\rangle = \sqrt{6}\sqrt{4+9+1}\cos\theta$

$2-6+1 = \sqrt{6}\sqrt{14}\cos\theta$

$$\boxed{\theta = \cos^{-1}\left(\frac{-3}{\sqrt{6}\sqrt{14}}\right)}$$

2. (20 points)



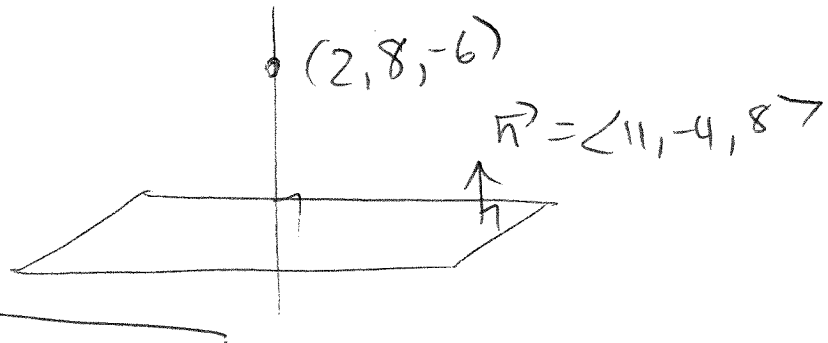
$\vec{PA} = \langle 2-1, 3-4, 4-7 \rangle = \langle 1, -1, -3 \rangle$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -3 \\ 1 & -1 & 2 \end{vmatrix} = \langle -2-(3), -(2+3), -1-(-1) \rangle$$

$$= \langle -5, -5, 0 \rangle$$

$$-5(x-2) - 5(y-3) + 0(z-4) = 0$$

3. (12 points)



$$x = 2 + 11t$$
$$y = 8 - 4t$$
$$z = -6 + 8t$$

5. (15 pts)

$$\vec{T_1} = \langle -\|\vec{T_1}\|\cos 60^\circ, \|\vec{T_1}\|\sin 60^\circ \rangle$$

$$\vec{T_1} + \vec{T_2} = \langle 0, 16 \rangle$$

$$\langle -T_1 \tfrac{1}{2}, T_1 \sqrt{3}/2 \rangle + \langle T_2 \cos 30^\circ, T_2 \sin 30^\circ \rangle$$

$$\langle -T_1 \tfrac{1}{2}, T_1 \sqrt{3}/2 \rangle + \langle T_2 \sqrt{3}/2, T_2 \tfrac{1}{2} \rangle = \langle 0, 16 \rangle$$

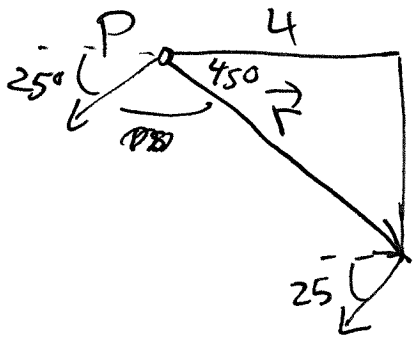
$$-T_1 \tfrac{1}{2} + T_2 \tfrac{\sqrt{3}}{2} = 0 \qquad T_1 = \sqrt{3}\, T_2$$

$$T_1 \tfrac{\sqrt{3}}{2} + T_2 \tfrac{1}{2} = 16$$

$$\tfrac{3}{2} T_2 + T_2 \tfrac{1}{2} = 16$$

$$2 T_2 = 16 \qquad \boxed{T_2 = 8} \qquad \boxed{T_1 = 8\sqrt{3}}$$

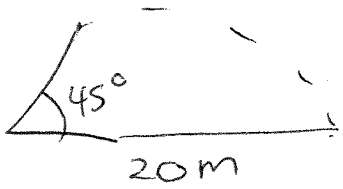
6. (13 points)



$$\|\vec{F}\| = \sqrt{16+16} = \sqrt{32}$$

$$\|\vec{r}\| = \boxed{24\sqrt{32}\sin(110^\circ)}$$

4. (20 points)



a) $\vec{a} = \langle 0, -10 \rangle$

$\vec{v} = \langle 0, -10t \rangle + \vec{C}$

$\vec{v}(0) = \langle v_0 \cos 45^\circ, v_0 \sin 45^\circ \rangle$

$= \langle v_0 \frac{\sqrt{2}}{2}, v_0 \frac{\sqrt{2}}{2} \rangle$

$$\boxed{\vec{v} = \langle v_0 \tfrac{\sqrt{2}}{2}, v_0 \tfrac{\sqrt{2}}{2} - 10t \rangle}$$

b) $\vec{r} = \langle v_0 \frac{\sqrt{2}}{2} t, v_0 \frac{\sqrt{2}}{2} t - 5t^2 \rangle + \cancel{\vec{d}}$

c) $v_0 \frac{\sqrt{2}}{2} t = 20$ $\qquad v_0 \frac{\sqrt{2}}{2} t - 5t^2 = 0$

$20 - 5t^2 = 0$

$4 = t^2 \qquad t = 2$

$v_0 \frac{\sqrt{2}}{2} \cdot 2 = 20$

$$\boxed{v_0 = 20/\sqrt{2}} = 10\sqrt{2}$$