MA 242 Test 1 Version 2

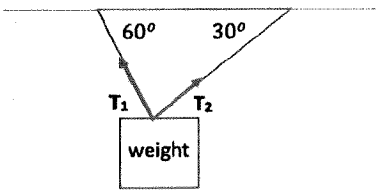
1. (20 points) Use vectors $\vec{a} = \langle 3, -1, 1 \rangle$ and $\vec{b} = \langle 2, 1, 1 \rangle$ to answer the following:
   a) Find a vector in the same direction as $\vec{a}$, but with magnitude 4
   b) Find the area of the parallelogram with adjacent edges $\vec{a}$ and $\vec{b}$
   c) Find the angle between $\vec{a}$ and $\vec{b}$

2. (20 points) Find an equation of the plane containing the point A(3,1,4) and the line x=2+t, y=5-2t, z= 3+t

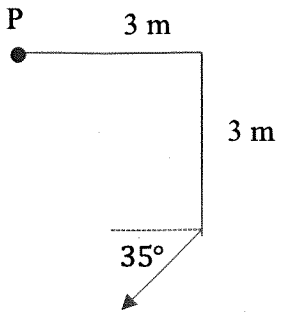
3. (12 points) Find parametric equations of the line through (7,4,1) and perpendicular to the plane 9x-4y+6z=3

4. (20 points) A ball is thrown from the ground at an angle of elevation of 45° above the horizontal with an initial speed v_0. The ball lands 20 m away. Use is $\vec{a}$ = <0,-10> for the acceleration due to gravity.
   a) Find the velocity vector $\vec{v}$ (Your answer can have v_0 in it)
   b) Find the position vector $\vec{r}$ (Your answer can have v_0 in it)
   c) Find the initial speed v_0

5. (15 points) A 20 lb weight is suspended from two cables as shown below.
   a) Write tension vector $\mathbf{T_1}$ in its component form.
   b) Find the magnitude of the tension in each cable



6. (13 points) Find the magnitude of the torque about point P if a 14 N force is applied as shown



C3 T1 V2 Solutions

1. (20 points)

a) $\sqrt{9+1+1}$

$\frac{4}{\sqrt{11}}\langle 3,-1,1\rangle = \boxed{\left\langle \frac{12}{\sqrt{11}}, \frac{-4}{\sqrt{11}}, \frac{4}{\sqrt{11}} \right\rangle}$

b) $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$

$= \langle -1-1, -(3-2), 3+2\rangle$

$= \langle -2, -1, 5\rangle$

$\|\vec{a}\times\vec{b}\| = \sqrt{4+1+25} = \boxed{\sqrt{30}}$

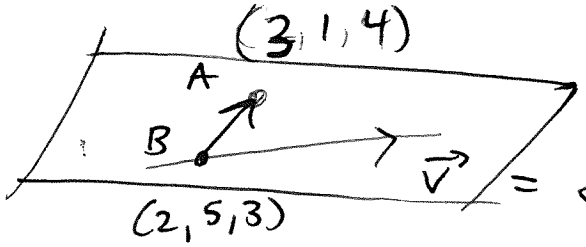
c) $\vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta$

$\langle 3,-1,1\rangle \cdot \langle 2,1,1\rangle = \sqrt{11}\sqrt{6}\cos\theta$

$6 - 1 + 1$

$\boxed{\theta = \cos^{-1}\left(\frac{6}{\sqrt{11}\sqrt{6}}\right)}$

2. (20 points)



$= \langle 1, -2, 1 \rangle$

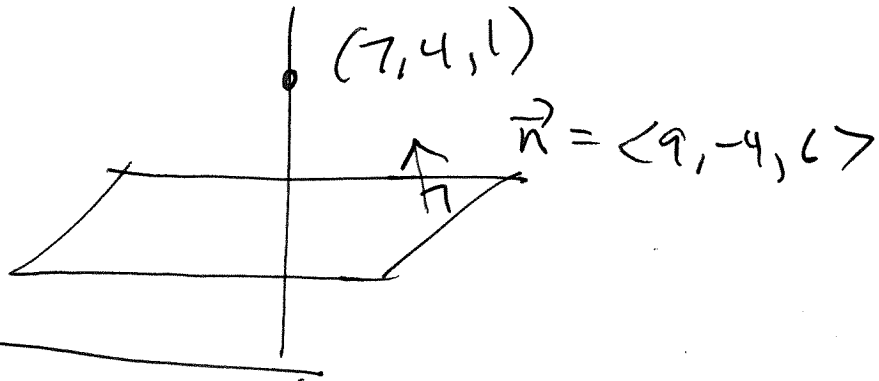
$\overrightarrow{BA} = \langle 3-2, 1-5, 4-3 \rangle = \langle 1, -4, 1 \rangle$

$\vec{n} = \overrightarrow{BA} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \langle -4+2, -(1-1), -2+4 \rangle$

$\equiv \langle -2, 0, 2 \rangle$

$-2(x-3) + 0(y-1) + 2(z-4) = 0$

3. (12 points)

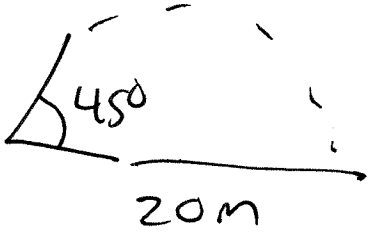


$x = 7 + 9t$

$y = 4 - 4t$

$z = 1 + 6t$

4. (20 points)



a) $\vec{a} = \langle 0, -10 \rangle$

$\vec{v} = \langle 0, -10t \rangle + \vec{C}$

$\vec{v}(0) = \langle v_0 \cos 45^\circ, v_0 \sin 45^\circ \rangle$

$= \langle v_0 \frac{\sqrt{2}}{2}, v_0 \frac{\sqrt{2}}{2} \rangle$

$\boxed{\vec{v} = \langle v_0 \frac{\sqrt{2}}{2}, v_0 \frac{\sqrt{2}}{2} - 10t \rangle}$

b) $\vec{r} = \langle v_0 \frac{\sqrt{2}}{2} t, v_0 \frac{\sqrt{2}}{2} t - 5t^2 \rangle + \vec{C}$

c) $v_0 \frac{\sqrt{2}}{2} t = 20$ $\qquad v_0 \frac{\sqrt{2}}{2} t - 5t^2 = 0$

$20 - 5t^2 = 0$

$4 = t^2$

$t = 2$

$\boxed{v_0 = \frac{20}{\sqrt{2}}}$

5. (15 pts)

a) $\vec{T_1} = \langle -\|\vec{T_1}\| \cos 60^\circ, \|\vec{T_1}\| \sin 60^\circ \rangle$

$= \langle -T_1 \tfrac{1}{2}, T_1 \tfrac{\sqrt{3}}{2} \rangle$

$\vec{T_2} = \langle T_2 \cos 30^\circ, T_2 \sin 30^\circ \rangle$

$= \langle T_2 \sqrt{3}/2, T_2 \tfrac{1}{2} \rangle$

$\vec{T_1} + \vec{T_2} = \langle 0, 20 \rangle$

$-T_1 \tfrac{1}{2} + T_2 \tfrac{\sqrt{3}}{2} = 0 \rightarrow T_1 = T_2\sqrt{3}$

$T_1 \tfrac{\sqrt{3}}{2} + T_2 \tfrac{1}{2} = 20$

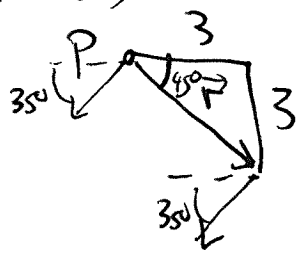
$T_2 \tfrac{3}{2} + T_2 \tfrac{1}{2} = 20$

$T_2 \tfrac{4}{2} = 20$

$T_2 = 10$

$T_1 = 10\sqrt{3}$

6. (13 points)



$\|\vec{r}\| = \sqrt{9+9} = \sqrt{18}$

$\|\vec{\tau}\| = \|\vec{F}\| \|\vec{r}\| \sin\theta = 14\sqrt{18} \sin 100^\circ$