MA 241 Test 2 Version 1 Remember: No work=No credit!

1. (15 points) Evaluate $\int \frac{2+5x}{x^3(x+2)}\,dx$

2. (12 points) Evaluate $\int_0^{\frac{\pi}{2}} \cos^3 x\,dx$

3. (17 points) Use **trig substitution** to evaluate $\int \frac{dx}{(5-4x-x^2)^{3/2}}$

Hint: For integrals with $\sqrt{a^2-u^2}$, let $u = a\sin\theta$

4. (26 points) Determine if the following integrals are convergent or divergent. Evaluate the convergent integrals.

a) $\int_0^{\infty} \frac{e^x\,dx}{e^{2x}+25}$

b) $\int_0^3 \frac{dx}{x-1}$

5. (14 points) $\int \tan^5 x \sec x\,dx$

6. (16 points) Use $\int_0^8 e^{x^2}\,dx$ to answer the following:

a) Use Simpson's Rule to approximate the integral with n=4

b) Find the upper bound of the error estimate using $|E_S| \le \frac{K(b-a)^5}{180n^4}$ where $|f^{(4)}(x)| \le K$ for $a \le x \le b$. Hint: $f^{(4)}(x) = 4e^{x^2}(4x^4+12x^2+3)$

C2 H T2 V1 Solutions

1. (15 points)

$$\int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+2}\,dx$$

$$Ax^2(x+2) + Bx(x+2) + C(x+2) + Dx^3 = 2+5x$$

$$Ax^3 + 2Ax^2 + Bx^2 + 2Bx + Cx + 2C + Dx^3 = 2+5x$$

$A+D=0$ $D=1$

$2A+B=0$ $A=-1$

$2B+C=5$ $B=2$

$2C=2$ $C=1$ 5

$$\int \frac{-1}{x} + \frac{2}{x^2} + \frac{1}{x^3} + \frac{1}{x+2}\,dx$$

$$\boxed{-\ln|x| - 2/x - \frac{1}{2}x^{-2} + \ln|x+2| + C}$$

2. (12 points) $\int_0^{\pi/2} \cos^2 x \cos x\,dx =$

$\int_0^{\pi/2} (1-\sin^2 x)\cos x\,dx$ $u=\sin x$ $du=\cos x$

$$\int_0^1 (1-u^2)\,du = u - \frac{1}{3}u^3 \Big|_0^1 = 1 - \frac{1}{3} = \boxed{\frac{2}{3}}$$

3. (17 points)

$$5-4x-x^2 = a^2-(x+b)^2 = a^2-x^2-2bx-b^2 = 5-4x-x^2$$

$$b=2 \qquad a^2-b^2=5$$
$$a^2=9$$

$$\int \frac{dx}{(9-(x+2)^2)^{3/2}} \qquad u=x+2 \quad du=dx$$

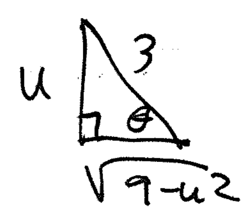
$$\int \frac{du}{(9-u^2)^{3/2}} \qquad u=3\sin\theta \quad du=3\cos\theta\, d\theta$$

$$\int \frac{3\cos\theta\, d\theta}{(9-9\sin^2\theta)^{3/2}}$$

$$\int \frac{3\cos\theta\, d\theta}{(9\cos^2\theta)^{3/2}} = \int \frac{3\cos\theta\, d\theta}{3^3\cos^3\theta}$$

$$\frac{u}{3}=\sin\theta$$

$$\int \frac{1}{9}\sec^2\theta\, d\theta = \frac{1}{9}\tan\theta + C$$



$$= \boxed{\frac{1}{9}\frac{u}{\sqrt{9-u^2}} + C}$$

4. (26 points)

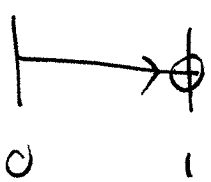
a) $\displaystyle\int_0^\infty \frac{e^x\,dx}{e^{2x}+25}$ $\qquad u=e^x \quad du=e^x\,dx$

$$\int_1^\infty \frac{du}{u^2+25} = \lim_{t\to\infty}\int_1^t \frac{du}{u^2+25}$$

$$=\lim_{t\to\infty} \frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)\Big|_1^t = \lim_{t\to\infty}\frac{1}{5}\left(\tan^{-1}\left(\frac{t}{5}\right)-\tan^{-1}\left(\frac{1}{5}\right)\right)$$

$$=\boxed{\frac{1}{5}\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{5}\right)\right)}$$ conv.

b) $\displaystyle\lim_{t\to1^-}\int_0^t \frac{dx}{x-1} = \lim_{t\to1^-} \ln|x-1|\Big|_0^t$



$$=\lim_{t\to1^-} \ln|t-1| - \cancel{\ln|-1|} \to -\infty$$

div.

so $\displaystyle\int_0^3 \frac{dx}{x-1}$ div

5. (14 points)

$$\int \tan^4 x \tan x \sec x \, dx$$

$$\int (1-\sec^2 x)^2 \tan x \sec x \, dx \qquad u = \sec x$$

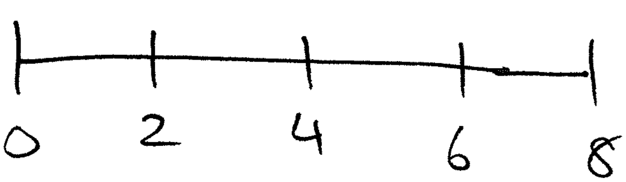
$$du = \sec x \tan x \, d$$

$$\int (1-u^2)^2 \, du$$

$$\int 1 - 2u^2 + u^4 \, du = u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$$

$$= \boxed{\sec x - \frac{2}{3}\sec^3 x + \frac{1}{5}\sec^5 x + C}$$

6. (16 points) $\Delta x = \dfrac{8-0}{4} = 2$



a) $S_4 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$

$= \frac{2}{3}\left[f(0) + 4f(2) + 2f(4) + 4f(6) + f(8)\right]$

$= \frac{2}{3}\left[1 + 4e^4 + 2e^{16} + 4e^{36} + e^{64}\right] \approx \int_0^8 e^{x^2} dx$

b) $|E_S| \leq \dfrac{(4e^{64}(4\cdot 8^4 + 12\cdot 8^2 + 3))(8-0)^5}{180\cdot 4^4}$