MA 242 Test 2 Version 1

1. (17 points) Find and classify all critical points of f(x,y)= $x^3 - 3xy + \frac{y^2}{2}$. Justify your answers as we have done in class.

2. (20 points) Use the function f(x,y)=$\tan(4x + y^3 - 9)$ to find the following:
   a) Find the directional derivative of f(x,y) at P(2,1) in the direction of $\vec{v} = \langle -1,3 \rangle$
   b) Find the **minimum** rate of change of f(x,y) at P(2,1)

3. (20 points) Find the global maximum and minimum values of f(x,y)=$xy - x$ over the region D bounded by the x-axis and $y = 13 - x^2$. Fully justify your answers as we have done in class.

4. (14 points) Use Lagrange multipliers to find the maximum and minimum values of the function f(x,y)= $2x + 6y$ subject to the given constraint, $x^2 + y^2 = 10$

5. (15 points) Find a vector equation for the tangent line of the curve with given parametric equations at the specified point:
   $x = ln(t), \ \ y = 2\sqrt{t}, \ \ z = t^2; \ \ (0,2,1)$

6. (14 points) Use the Calculus 3 Chain Rule to first write a formula for $\frac{\partial z}{\partial t}$ and then find it given z=e^{2x+3y}, where x=$\frac{s}{t}$, y=t^s (y=t raised to the s) when s=2 and t=1.

C3T2V1 Solutions

1. (17 points)

$f_x = 3x^2 - 3y = 0 \qquad y = x^2$

$f_y = -3x + y = 0 \qquad y = 3x$

$3x = x^2$

$x = 0, \; x = 3$

$y = 0 \quad y = 9$

$f_{xx} = 6x \qquad f_{yy} = 1$

$f_{xy} = -3$

$D = f_{xx} f_{yy} - [f_{xy}]^2$

$D = 6x \cdot 1 - [-3]^2$

$D(0,0) = -9 < 0$ saddle pt

$D(3,9) = 18 - 9 > 0$

$f_{xx}(3,9) = 18 > 0 \qquad (3,9) = \text{local min}$

2. (20 points)

a) $\nabla f = \langle 4\sec^2(4x+y^3-9), 3y^2\sec^2(4x+y^3-9)\rangle$

$\nabla f(2,1) = \langle 4\sec^2(8+1-9), 3\sec^2 0\rangle$

$= \langle 4, 3\rangle$

$\hat{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{\langle -1, 3\rangle}{\sqrt{1+9}} = \left\langle \dfrac{-1}{\sqrt{10}}, \dfrac{3}{\sqrt{10}}\right\rangle$

$D_{\hat{u}} f = \nabla f \circ \hat{u} = \langle 4, 3\rangle \cdot \left\langle \dfrac{-1}{\sqrt{10}}, \dfrac{3}{\sqrt{10}}\right\rangle$

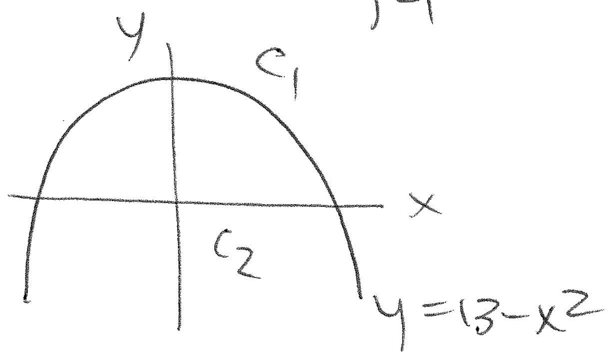
$= \dfrac{-4}{\sqrt{10}} + \dfrac{9}{\sqrt{10}} = \boxed{\dfrac{5}{\sqrt{10}}}$

b) $-\|\nabla f\| = -\sqrt{4^2+3^2} = -5$

3. (20 points) $f = xy - x$

$f_x = y - 1 = 0 \quad f_y = x = 0$

$y = 1 \quad x = 0$



Candidates

$f(0,1) = 0$

$f(2,9) = 18 - 2 = 16$

$f(-2,9) = -18 + 2 = -16$

$f(\sqrt{13}, 0) = -\sqrt{13}$

$f(-\sqrt{13}, 0) = \sqrt{13}$

C_1: $f(x, 13-x^2) = x(13-x^2) - x$

$= 13x - x^3 - x$

$= 12x - x^3$

$f_x(x, 13-x^2) = 12 - 3x^2 = 0$

$4 = x^2$

$x = \pm 2$

C_2: $y = 0$

$f(x,0) = -x$

$f_x(x,0) = -1 \neq 0$

Global max $= 16$

Global min $= -16$

4. (14 points)

$$\nabla f = \langle 2, 6 \rangle = \lambda \langle 2x, 2y \rangle$$

$2 = \lambda 2x \qquad 6 = \lambda 2y$

$1 = \lambda x \qquad 3 = \lambda y$

$\lambda \neq 0$

$x = \frac{1}{\lambda} \qquad y = \frac{3}{\lambda}$

$x^2 + y^2 = 10$

$\left(\frac{1}{\lambda}\right)^2 + \left(\frac{3}{\lambda}\right)^2 = 10$

$\frac{10}{\lambda^2} = 10$

$\lambda^2 = 1 \qquad \lambda = 1: \ (1, 3) \quad f(1,3) = 2 + 18 = 20$

$\lambda = -1: \ (-1, -3) \quad f(-1,-3) = -20$

5. (15 points)

$x' = \frac{1}{t} \qquad y' = \frac{1}{\sqrt{t}} \qquad z' = 2t \qquad t = 1$

$x'(1) = 1 \qquad y'(1) = 1 \qquad z'(1) = 2$

$$\boxed{\vec{r} = \langle 0, 2, 1 \rangle + \langle 1, 1, 2 \rangle t}$$

6. (14 points)

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = \left(2e^{2x+3y}\right)\left(\frac{-s}{t^2}\right) + \left(3e^{2x+3y}\right)\left(st^{s-1}\right)$$

$s=2,\ t=1 \rightarrow \quad x=2,\ y=1$

$$\frac{\partial z}{\partial t} = (2e^7)(-2) + 3e^7(2) = \boxed{e^7}$$