MA 242 Test 2 Version 2

1. (17 points) Find and classify all critical points of f(x,y)= $x^3 - 3xy + \frac{y^2}{2}$. Justify your answers as we have done in class.

2. (20 points) Use the function f(x,y)=tan$(2x + y^3 - 7)$ to find the following:
   a) Find the directional derivative of f(x,y) at P(3,1) in the direction of $\vec{v} = \langle -2,1 \rangle$
   b) Find the **minimum** rate of change of f(x,y) at P(3,1)

3. (20 points) Find the global maximum and minimum values of f(x,y)=$xy - x$ over the region D bounded by the x-axis and $y = 13 - x^2$. Fully justify your answers as we have done in class.

4. (14 points) Use Lagrange multipliers to find the maximum and minimum values of the function f(x,y)= $4x + 2y$ subject to the given constraint, $x^2 + y^2 = 5$

5. (15 points) Find a vector equation for the tangent line of the curve with given parametric equations at the specified point:
   $x = 2\sqrt{t}, \quad y = ln(t), \quad z = t^3; \quad (2,0,1)$

6. (14 points) Use the Calculus 3 Chain Rule to first write a formula for $\frac{\partial z}{\partial t}$ and then find it given z=e^{3x+6y}, where x=$\frac{s}{t}$, y=t^s (y=t raised to the s) when s=2 and t=1.

C3T2V2 Solutions

1. (17 points)

$f_x = 3x^2 - 3y = 0 \quad y = x^2$

$f_y = -3x + y = 0 \quad y = 3x$

$3x = x^2$

$x = 0, x = 3$

$\swarrow \qquad \swarrow$

$y = 0 \qquad y = 9$

$f_{xx} = 6x \quad f_{yy} = 1 \quad f_{xy} = -3$

$D = f_{xx} f_{yy} - [f_{xy}]^2$

$D = 6x - [-3]^2 = 6x - 9$

$D(0,0) = -9 < 0$ saddle pt

$D(3,9) = 18 - 9 > 0$

$f_{xx}(3,9) = 18 > 0 \quad \cup$

$(3,9)$ local min

2. (20 points) $\nabla f = \langle 2\sec^2(2x+y^3-7), 3y^2\sec^2(2x+y^3-7)\rangle$

a) $\nabla f(3,1) = \langle 2\sec^2(6+1-7), 3\sec^2 0\rangle$

$= \langle 2, 3\rangle$

$\hat{u} = \frac{\vec{v}}{\|\vec{v}\|} = \langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle$

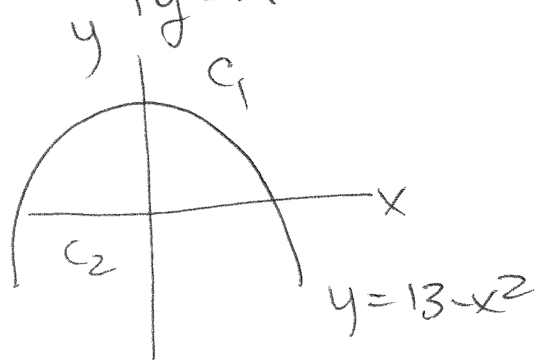
$D_{\hat{u}} f = \nabla f \cdot \hat{u} = \langle 2, 3\rangle \cdot \langle -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle = \frac{-4}{\sqrt{5}} + \frac{3}{\sqrt{5}} = \boxed{\frac{-1}{\sqrt{5}}}$

b) $-\|\nabla f(3,1)\| = -\sqrt{4+9} = \boxed{-\sqrt{13}}$

3. (20 points) $f = xy - x$

$f_x = y - 1 = 0$

$f_y = x = 0$



candidates

$f(0,1) = 0$

$f(2,9) = 18 - 2 = 16$

$f(-2,9) = -18 + 2 = -16$

$f(\sqrt{13}, 0) = -\sqrt{13}$

$f(-\sqrt{13}, 0) = \sqrt{13}$

C_1: $f(x, 13-x^2) = x(13-x^2) - x = 13x - x^3 - x$

$= 12x - x^3$

$f_x(x, 13-x^2) = 12 - 3x^2 = 0$

$x^2 = 4$

$x = \pm 2$

C_2: $y = 0$

$f(x,0) = -x$

$f_x(x,0) = -1 \neq 0$

No candidates

Endpts $(\pm\sqrt{13}, 0)$

$$\boxed{\begin{array}{l} \text{Global max} = 16 \\ \text{Global min} = -16 \end{array}}$$

5. (15 points) $2=2\sqrt{t}$, $0=\ln t$, $1=t^3 \rightarrow t=1$

$x'=\frac{1}{\sqrt{t}}$ $y'=\frac{1}{t}$ $z'=3t^2$

$x'(1)=1$ $y'(1)=1$ $z'(1)=3$

$$\boxed{\vec{r}=\langle 2,0,1\rangle+\langle 1,1,3\rangle t}$$

6. (14 points)

$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial z}{\partial t}=\left(3e^{3x+6y}\right)\left(\frac{-s}{t^2}\right)+\left(6e^{3x+6y}\right)\left(st^{s-1}\right)$$

$s=2, t=1 \rightarrow x=2, y=1$

$$\frac{\partial z}{\partial t}=3e^{12}(-2)+6e^{12}(2)=\boxed{6e^{12}}$$

4. (14 points)

$$\nabla f = \lambda \nabla g$$

$$\langle 4, 2 \rangle = \lambda \langle 2x, 2y \rangle$$

$4 = \lambda 2x \qquad 2 = \lambda 2y$

$2 = \lambda x \qquad 1 = \lambda y$

$\lambda \neq 0$

$\frac{2}{\lambda} = x \qquad \frac{1}{\lambda} = y$

$$x^2 + y^2 = 5$$

$$\left(\frac{2}{\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 = 5$$

$\frac{5}{\lambda^2} = 5 \qquad \lambda^2 = \pm 1$

$\lambda = 1$: $(2, 1)$

$\lambda = -1$: $(-2, -1)$

$f(2,1) = 8 + 2 = 10 \leftarrow$ max

$f(-2,-1) = -8 - 2 = -10 \leftarrow$ min