Name: ________________________ MA 341 Test 1 Version 1

1. (14 points) Solve the Initial Value Problem (IVP): $\frac{dy}{dx} = \frac{\sec^2 x}{e^{2y}}$, $y(0) = 0$
   Write your answer with y as an explicit function of x if possible.

2. (22 points) Use the IVP $\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos(x)}{x^2}$, $y\left(\frac{\pi}{2}\right) = 0$ to answer the following:
   a) Solve the IVP. Write your answer with y as an explicit function of x if possible.
   b) Does the Existence and Uniqueness Theorem guarantee that this is a unique solution? Justify your answer.

3. (12 points) A large tank initially contains 65 liters of brine in which 3 kg of salt has been dissolved. Brine solution flows into the tank at a rate of 9 L/min. The well-mixed solution leaves the tank at a rate of 7 L/min. If the concentration of salt in the brine entering the tank is 0.5 kg/L and x(t) is the amount of salt in the tank at time t, formulate the IVP that describes this system. **Do not solve the differential equation.**

4. (13 points) Use $(1 + 3x^2 \ln(y))dx + \left(\frac{x^3}{y} + \frac{1}{\sqrt{y}}\right) dy = 0$ to answer the following:
   a) Show that this is an exact differential equation
   b) Find its implicit general solution

5. (12 points) Solve the Boundary Value Problem (BVP):
   $y'' - 16y' + 64y = 0,\ y(0) = 3, y(1) = 0$

6. (14 points) Use the differential equation $\frac{dy}{dt} = (6 - y)(y - 3)$ to answer the following:
   a) Sketch its phase line and classify its equilibria as we have done in class
   b) If y(0)=3, without solving the differential equation, find y(t) in general.
   c) Use the phase line to determine the asymptotic behavior as t→ ∞ of the solution through y(0)= 4
   d) Is this differential equation separable, linear, both, or neither? No explanation is needed.

7. (13 points) Determine if $x^2 - \sin(x + y) = 2$ is a solution to $\frac{dy}{dx} = 2x\sec(x + y) - 1$

MA 341 T1 V1 Solutions

1. (14 points)

$$\int e^{2y}\,dy = \int \sec^2 x\,dx$$

$$\tfrac{1}{2}e^{2y} = \tan x + C$$

$$e^{2y} = 2\tan x + C_1$$

$$2y = \ln(2\tan x + C_1)$$

$$y = \tfrac{1}{2}\ln(2\tan x + C_1)$$

$$y(0) = 0 = \tfrac{1}{2}\ln(2\cancel{\tan 0}^{\,0} + C_1)$$

$$\boxed{y = \tfrac{1}{2}\ln(2\tan x + 1)}$$

2. (22 points)

a) $\mu(x) = e^{\int \frac{2}{x} dx} = e^{2\ln x} = x^2$

$\frac{d}{dx}\left[\mu(x) y\right] = \mu \overline{Q}$

$\frac{d}{dx}\left[x^2 y\right] = \cos x$

$x^2 y = \sin x + C$

$y = \frac{\sin x + C}{x^2}$

$y(\frac{\pi}{2}) = 0 = \frac{\sin\frac{\pi}{2} + C}{(\frac{\pi}{2})^2}$ $\quad C = -1$

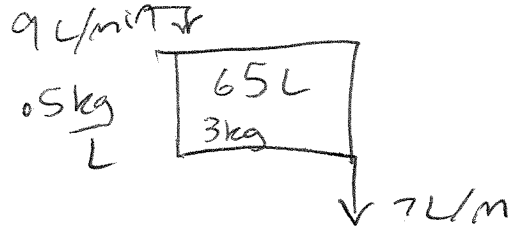
$$\boxed{y = \frac{\sin x - 1}{x^2}}$$

b) $f = \frac{dy}{dx} = \frac{\cos x}{x^2} - \frac{2y}{x}$ $\quad$ f is cont at & around $(\frac{\pi}{2}, 0)$

$\frac{\partial f}{\partial y} = \frac{-2}{x}$ $\quad$ $\frac{\partial f}{\partial y}$ cont at & around $(\frac{\pi}{2}, 0)$

Yes.

3. (12 points)



$$\frac{dx}{dt} = F_i C_i - F_o C_o$$

$$\boxed{\frac{dx}{dt} = 9\left(\tfrac{1}{2}\right) - 7\left(\frac{x}{65+2t}\right) \qquad x(0) = 3}$$

4. (13 points)

a) $M = 1 + 3x^2 \ln y \qquad N = \frac{x^3}{y} + \frac{1}{\sqrt{y}}$

$M_y = \frac{3x^2}{y} = N_x = \frac{3x^2}{y}$ ✓

b)

$F_x = 1 + 3x^2 \ln y$

$F = x + x^3 \ln y + g(y)$

$F_y = 0 + \frac{x^3}{y} + g'(y) = \frac{x^3}{y} + \frac{1}{\sqrt{y}}$

$g'(y) = \frac{1}{\sqrt{\cancel{x}\,y}} = y^{-1/2}$

$g(y) = 2\sqrt{y}$

$$\boxed{x + x^3 \ln y + 2\sqrt{y} = C}$$

5. (12 points)

$r^2 - 16r + 64 = 0$

$(r-8)^2 = 0$

$y = c_1 e^{8t} + c_2 t e^{8t}$

$y(0) = 3 = c_1$

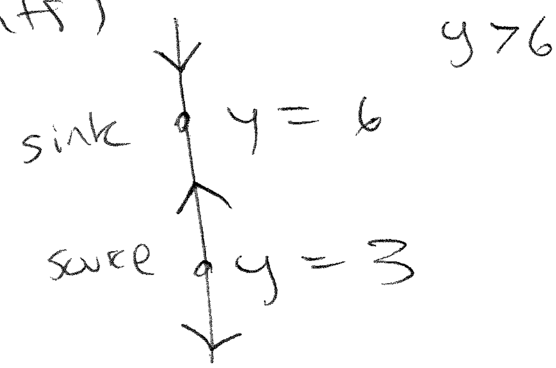
$y = 3e^{8t} + c_2 t e^{8t}$

$y(1) = 0 = c_1 e^8 + c_2 e^8 \qquad c_1 = -c_2$

$\boxed{y = 3e^{8t} - 3te^{8t}}$

6. (14 points)

a)



b) $y(t) = 3$

c) $y \to 6$

d) separable

7. (13 points)

$$x^2 - \sin(x+y) = 2$$

$$2x - \cos(x+y)\left(1 + \frac{dy}{dx}\right) = 0$$

$$2x = \cos(x+y)\left(1 + \frac{dy}{dx}\right)$$

$$= \cos(x+y) + \cos(x+y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x - \cos(x+y)}{\cos(x+y)} \cdot \frac{\sec(x+y)}{\sec(x+y)}$$

$$= \frac{2x\sec(x+y) - 1}{1}$$ ✓

Yes