MA 242 Test 3 Version 2

Calculators are not allowed

1. (12 points) Set up the double integral needed to find the volume under $z = x^2 + 3y$ and above the region in the xy - plane bounded by the curves $y = 4$, $y = \ln x$, and $x = 1$ **Do not evaluate.**

2. (12 points) Evaluate by first switching the order of integration $\int_0^9 \int_{\sqrt{x}}^3 \frac{e^{-y}}{y^2}\, dy\, dx$

3. (25 points) Spherical Coordinates

a) F is the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and beneath the sphere $x^2 + y^2 + z^2 = 9$, set up the integral needed to find the volume of F using spherical coordinates. **Do not evaluate.**

b) Evaluate $\iiint_F z^2\, dV$ if F is the solid bounded by the xz - plane and the hemispheres $y = \sqrt{4 - x^2 - z^2}$ and $y = \sqrt{1 - x^2 - z^2}$

4. (14 points) Set up the integral needed to find the mass of the solid tetrahedron bounded by the plane passing through the points (3,0,0), (0,6,0), and (0,0,12), the yz - plane, the xz - plane, and the plane $z = 2$ with density $\sigma(x,y,z) = x^2 z$ **Do not evaluate.**

5. (23 points) Use $f(x,y) = x$ over the region D bounded by the x - axis, the y - axis,and $y = 6 - 3x$ to answer the following:

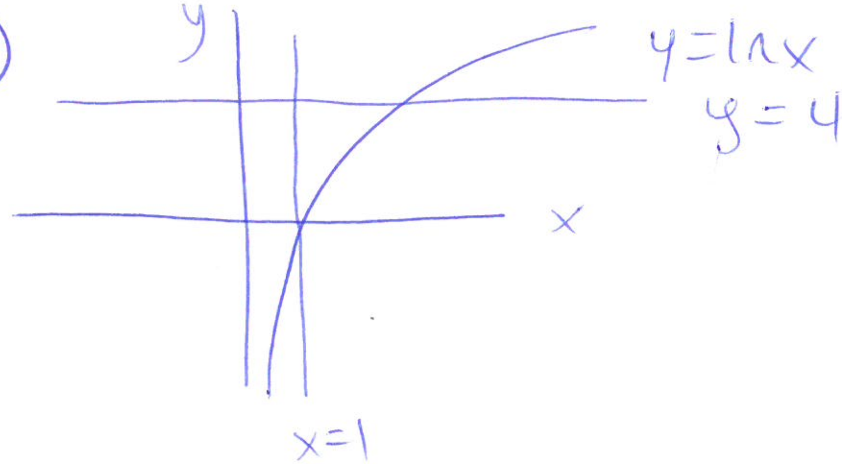
a) Set up the integral $\iint_D f(x,y)\, dA$ in polar coordinates. **Do not evaluate.**

b) Find the average value of $f(x,y)$ over the region D using rectangular coordinates

6. (14 points) Find the moment of inertia, $I_z = \iiint_F (x^2 + y^2)\, dV$, if F is bounded by the paraboloid $z = 5x^2 + 5y^2$ and the plane $z = 20$

MA 242 T3 Solutions

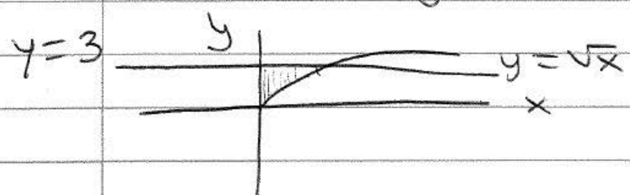
1. (12 points)

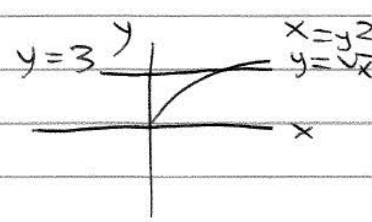


$$\int_1^{e^4}\int_{\ln x}^{4} x^3+3y\,dy\,dx$$

2. (12 pts)

$$\int_0^9\int_{\sqrt{x}}^3 \frac{e^{-y}}{y^2}\,dy\,dx$$





$$\int_0^3\int_0^{y^2} \frac{e^{-y}}{y^2}\,dx\,dy$$

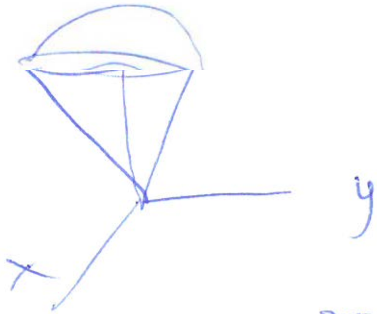
$$= \int_0^3 \frac{e^{-y}}{y^2}\, x \Big|_0^{y^2} dy$$

$$= \int_0^3 \frac{e^{-y}}{\cancel{y^2}}\, \cancel{y^2}\, dy = -e^{-y}\Big|_0^3$$

$$= \boxed{-e^{-3}+1}$$

3. (25 pts)

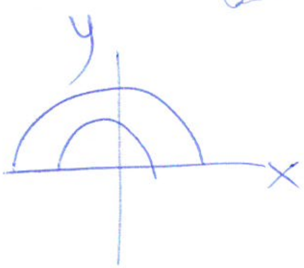
a)



$$\iiint_E 1\,dV = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{3} 1\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

b)

$$\int_0^{\pi}\int_0^{\pi}\int_1^{2} \rho^2\cos^2\phi\;\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$



$$\pi\int_0^{\pi}\cos^2\phi\sin\phi\,d\phi\int_1^2\rho^4\,d\rho$$

$u=\cos\phi$

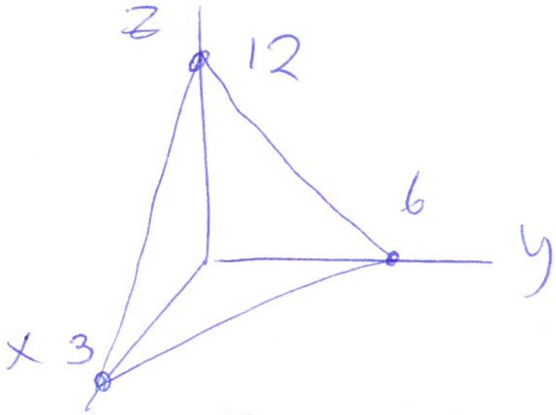
$du=-\sin\phi$

$$\pi\int_1^{-1}-u^2\,du\quad \frac{1}{5}\rho^5\Big|_1^2$$

$$\pi\int_{-1}^{1}u^2\,du\quad\frac{1}{5}(2^5-1^5)$$

$$\boxed{\frac{2\pi}{3}\cdot\frac{1}{5}(31)}$$

4. (14 pts)



$z = ax + by + c \qquad z = 12 + ax + by$

$(3,0,0): \quad 0 = 12 + 3a \qquad a = -4$

$(0,6,0): \quad 0 = 12 + 6b \qquad b = -2$

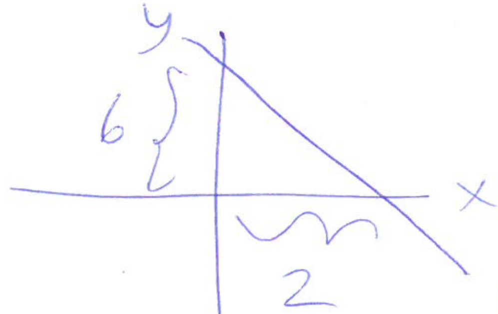
$z = 12 - 4x - 2y$

$12 - 4x - 2y = 2$

$10 - 4x = 2y \rightarrow y = 5 - 2x$

$$\boxed{\int_0^{\frac{5}{2}} \int_0^{5-2x} \int_2^{12-4x-2y} x^2 z \, dz \, dy \, dx}$$

5. (23pts)



$y = 6-3x$

a) $\displaystyle\int_0^{\pi/2}\int_0^{\frac{6}{\sin\theta+3\cos\theta}} r\cos\theta \, r\,dr\,d\theta$

$r\sin\theta = 6-3r\cos\theta$

$r\sin\theta + 3r\cos\theta = 6$

$r = \dfrac{6}{\sin\theta + 3\cos\theta}$

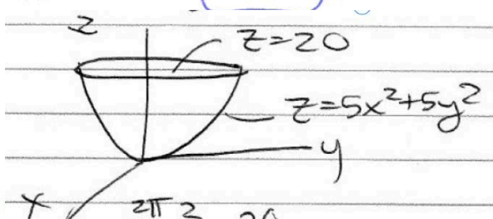
b) $\dfrac{\displaystyle\int_0^2\int_0^{6-3x} x\,dy\,dx}{\frac{1}{2}(6)(2)} = \dfrac{1}{6}\displaystyle\int_0^2 x(6-3x)\,dx$

$\dfrac{1}{6}\displaystyle\int_0^2 6x-3x^2\,dx$

$\dfrac{1}{6}\left[3x^2-x^3\right]_0^2$

$= \dfrac{1}{6}\left[12-8\right] = \boxed{\dfrac{4}{6}}$

6. (14pts)



$\displaystyle\int_0^{2\pi}\int_0^{2}\int_{5r^2}^{20} r^2\,dz\,r\,dr\,d\theta$

$20 = 5r^2$

$2\pi\displaystyle\int_0^2 r^3(20-5r^2)\,dr$

$2\pi\displaystyle\int_0^2 20r^3-5r^5\,dr = 2\pi\left[5r^4-\frac{5}{6}r^6\right]_0^2$

$2\pi\left[5\cdot 2^4-\frac{5}{6}\cdot 2^6\right]$