MA 241 Test 3 Version 1

1. (14 points) Solve the Initial Value Problem (IVP): $y'' - 6y' + 10y=0$; y(0)=2, $y'(0)=0$

2. (18 points) Solve the Initial Value Problem (IVP): $y'' - 4y' = 20e^{4x}$; y(0)=5, $y'(0)=1$

3. (17 points) An integral equation is an equation containing an unknown function y(x) and an integral involving y(x). Find an explicit solution to the given integral equation
$$y(x) = \int_0^x \frac{\sec^2 t}{e^{2y(t)}}\, dt$$

4. (13 points) A 64 lb weight attached to a spring stretches it 6 inches before coming to a rest at equilibrium. The damping constant is 1 lb - sec/ft. At time t = 0, the spring is compressed 2 inches and released. If x(t) is the position of the mass at time t, use 32 ft/s^2 for the gravitational constant and formulate the IVP that describes this system **(Do not solve it)**

5. (13 points) Use Euler's method with a step size of 0.1 to estimate y(20.1) and y(20.2), if $y'=y^2 + x;\ y(20) = 0$. Clearly label your answers.

6. (12 points) A large tank initially contains 300 L of brine in which 4 kg of salt has been dissolved. At time t=0, pure water enters the tank at a rate of 9 L/min. The well-mixed solution leaves the tank at rate of 6 L/min. If y(t) is the amount of salt in the tank at time t, formulate the IVP that describes this system. **Do not solve it.**

7. (13 points) Find the orthogonal trajectories of $y = k\sqrt{x}$. What are they a family of?

C2 T3 V1 Solutions

1. (14 points)

$$r^2 - 6r + 10 = 0$$

$$r = \frac{6 \pm \sqrt{36-40}}{2} = \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = 3 \pm i$$

$$y = e^{3t}\left[c_1 \cos t + c_2 \sin t\right]$$

$$y(0) = 2 = e^0\left[c_1 \cos 0 + \cancel{c_2 \sin 0}\right]$$

$$c_1 = 2$$

$$y = e^{3t}\left[2\cos t + c_2 \sin t\right]$$

$$y' = 3e^{3t}\left[2\cos t + c_2 \sin t\right] + e^{3t}\left[-2\sin t + c_2 \cos t\right]$$

$$y'(0) = 0 = 3[2] + c_2 \qquad c_2 = -6$$

$$\boxed{y = e^{3t}\left[2\cos t - 6\sin t\right]}$$

2. (18 points)

$$r^2 - 4r = 0$$

$$r(r-4) = 0$$

$$y_c = C_1 e^{0*} + C_2 e^{4*}$$

$$y_c = C_1 + C_2 e^{4*}$$

$$y_p = A e^{4x} x$$

$$y'_p = 4Ae^{4x}x + Ae^{4x}$$

$$y''_p = 16Ae^{4x}x + 4Ae^{4x} \cdot 1 + 4Ae^{4x}$$

$$y'' - 4y' = 20e^{4x}$$

$$\cancel{16Ae^{4x}x} + 8Ae^{4x} - 4(\cancel{4Ae^{4x}x} + Ae^{4x}) = 20e^{4x}$$

$$8A - 4A = 20$$

$$4A = 20 \qquad A = 5$$

$$y = C_1 + C_2 e^{4x} + 5e^{4x}x$$

$$y(0) = C_1 + C_2 e^0 + \cancel{5e^0 \cdot 0} = 5$$

$$y' = 0 + 4C_2 e^{4x} + 5e^{4x} + 20e^{4x}x$$

$$y'(0) = 1 = 4C_2 + 5 + 0 \qquad C_2 = -1 \qquad C_1 = 6$$

$$\boxed{y = 6 - e^{4x} + 5e^{4x}x}$$

3. (17 points)

$$\frac{dy}{dx} = \frac{\sec^2 x}{e^{2y}} \qquad y(0)=0$$

$$\int e^{2y}\,dy = \int \sec^2 x\,dx$$

$$\frac{1}{2}e^{2y} = \tan x + C$$

$$e^{2y} = 2\tan x + C_1$$

$$2y = \ln(2\tan x + C_1)$$

$$y = \frac{1}{2}\ln(2\tan x + C_1)$$

$$y(0) = 0 = \frac{1}{2}\ln(2\tan 0 + C_1)$$

$$C_1 = 1$$

$$\boxed{y = \frac{1}{2}\ln(2\tan x + 1)}$$

4. (13 points)

$$mx'' + bx' + kx = F_{ext}$$

$W = mg$

$64 = m(32)$

$m = 2$ $b = 1$

$F = kx$

$64 = k\left(\frac{1}{2}\right)$

$k = 128$

$$\boxed{2x'' + x' + 128x = 0 \qquad x(0) = -2/12 \quad x'(0) = 0}$$

5. (13 points)

$x_0 = 20$ $y_0 = 0$

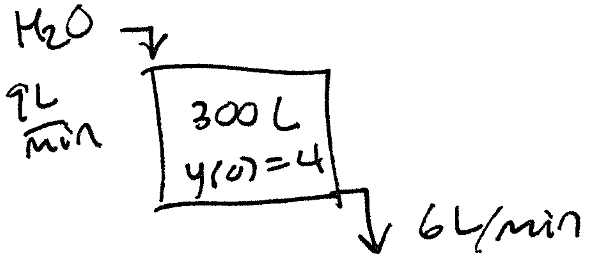
$x_1 = 20.1$

$x_2 = 20.2$

$$y(20.1) \approx y_1 = y_0 + f(x_0, y_0)h$$
$$= 0 + f(20,0)(.1)$$
$$= [0^2 + 20](.1) = 2$$

$$y(20.2) \approx y_2 = y_1 + f(x_1, y_1)h$$
$$= 2 + f(20.1, 2)(.1)$$
$$= 2 + [2^2 + 20.1](.1)$$
$$= 2 + [24.1](.1)$$
$$= 4.41$$

6. (12 points)



$$\frac{dy}{dt} = F_iC_i - F_oC_o$$

$$\boxed{\begin{aligned} \frac{dy}{dt} &= 9(0) - 6\left(\frac{y}{300+3t}\right) \\ y(0) &= 4 \end{aligned}}$$

7. (13 points)

$$\frac{dy}{dx} = \frac{k}{2\sqrt{x}}$$

$\perp$: $\frac{dy}{dx} = \frac{-2\sqrt{x}}{k}$

k: $k = y/\sqrt{x}$

$$\frac{dy}{dx} = \frac{-2\sqrt{x}}{y/\sqrt{x}} = \frac{-2x}{y}$$

$$\int y\,dy = \int -2x\,dx$$

$$\frac{1}{2}y^2 = -x^2 + C$$

$$\boxed{\frac{1}{2}y^2 + x^2 = C}$$

Family of ellipses