MA 242 Test 3 Version 1

1. (10 points) Sketch the region of integration and change the order of integration $\int_0^4 \int_0^{y/4} f(x,y)\,dx\,dy$

2. (20 points) **Set up** the integral needed to find the mass of the solid F bounded by the paraboloids $z = x^2 + y^2$ and $z = 12 - 2x^2 - 2y^2$ with density $\sigma(x,y,z) = e^{yz}$ using cylindrical coordinates. **Do not evaluate.**

3. (18 points) Find the average value of f(x,y)= y$\sqrt{x^2+y^2}$ over the lamina bounded by $x^2 + y^2 = 9$, where $y \geq 0$

4. (22 points) a) Evaluate $\iiint_F z\,dV$, where F is the solid that lies above the xy-plane, under the hemisphere z=$\sqrt{4 - x^2 - y^2}$, and below the cone z=$\sqrt{x^2+y^2}$

   b) **In general**, what is x in spherical coordinates?

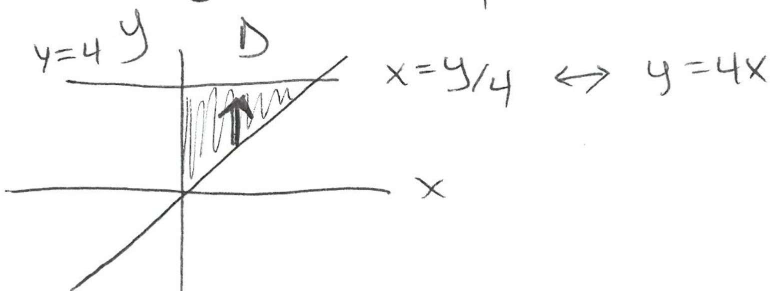
5. (15 points) Use a double integral to find the volume of the solid bounded by the cylinder $z = 11 - x^2$, the planes $z = 2$, y=5, and the xz-plane.

6. (15 points) **Set up** the iterated integral $\iiint_F ln(x+y+z)\,dV$ if F is the solid tetrahedron with vertices (0,0,0), (2,0,0), (0,10,0), (0,0,10). **Do not evaluate.**

C3 T3 V1 Solutions

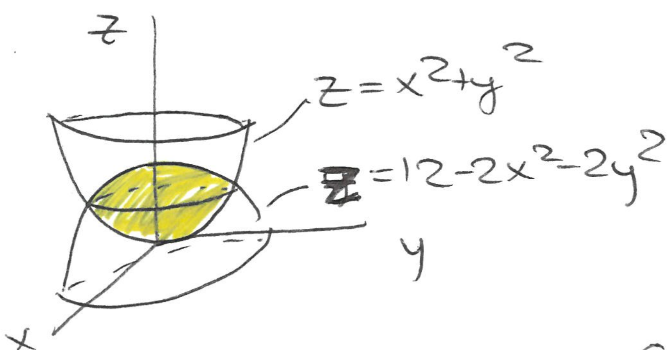
1. (10 points)

$0 \le x \le y/4$



$$\boxed{\int_0^1 \int_{4x}^4 f(x,y)\,dy\,dx}$$

2. (20 points)



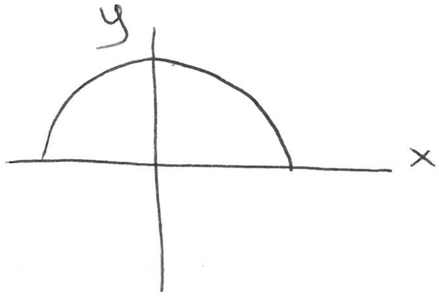
$$m = \iiint_E \sigma\, dV = \int_0^{2\pi}\int_0^2\int_{r^2}^{12-2r^2} e^{r\sin\theta\, z}\,dz\,r\,dr\,d\theta$$

$12-2r^2=r^2$

$12=3r^2$

$4=r^2$

3. (18 points)



$$f_{ave} = \frac{\iint_D f \, dA}{\iint_D 1 \, dA}$$

$$= \frac{\int_0^{\pi} \int_0^{3} r\sin\theta \sqrt{r^2} \, r\,dr\,d\theta}{\frac{1}{2}\pi(3)^2} = \frac{\int_0^{\pi} \sin\theta \, d\theta \int_0^{3} r^3 \, dr}{9\pi/2}$$

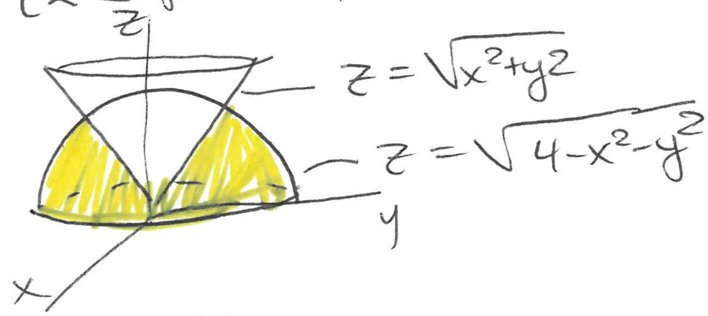
or

$$\int_0^{\pi} \int_0^{3} 1 \, r\,dr\,d\theta$$

$$= \frac{-\cos\theta \Big|_0^{\pi} \; \frac{1}{4} r^4 \Big|_0^{3}}{9\pi/2}$$

$$= \frac{(-\cos\pi + \cos 0)\frac{1}{4} \cdot 3^4}{9\pi/2} = \boxed{\frac{2/4 \; 3^4}{9\pi/2}} = \frac{9}{\pi}$$

4. (22 points)



$$\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^2 \rho\cos\phi\, \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$2\pi \int_{\pi/4}^{\pi/2} \cos\phi\sin\phi\, d\phi \int_0^2 \rho^3\, d\rho$$

$u=\sin\phi$

$du=\cos\phi\, d\phi$

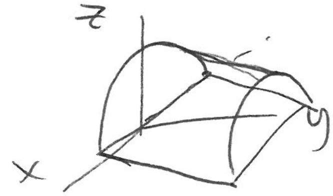
$$2\pi \int_{\sqrt{2}/2}^{1} u\, du \quad \frac{1}{4}\rho^4\Big|_0^2$$

$$2\pi \; \frac{1}{2}u^2\Big|_{\sqrt{2}/2}^{1} \; \frac{1}{4}\cdot 2^4$$

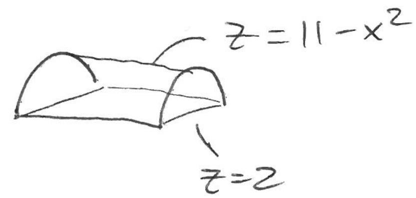
$$\pi\left[1-\frac{1}{2}\right]\cdot 4 = \boxed{2\pi}$$

b) $x=\rho\sin\phi\cos\theta$

5. (15 points)



$$\int_{-3}^{3}\int_{0}^{5} 11-x^2-2 \, dy \, dx$$



$11-x^2=2$

$9=x^2$

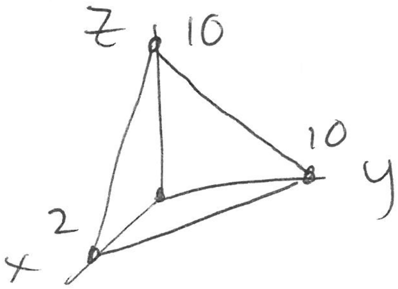
$$\int_{-3}^{3}\int_{0}^{5} 9-x^2 \, dy \, dx$$

$$\int_{-3}^{3} (9-x^2)y\Big|_0^5 \, dx = \int_{-3}^{3} 5(9-x^2) \, dx$$

$$= 10\int_0^3 9-x^2 \, dx = 10\left[9x-\tfrac{1}{3}x^3\right]_0^3$$

$$= 10\left[27-9\right] = \boxed{180}$$

6. (15 points)



$z=ax+by+c$

$(0,0,10)$: $10=c$

$z=10+ax+by$

$(2,0,0)$: $0=2a+10$ $a=-5$

$(0,10,0)$: $0=10b+10$ $b=-1$

$z=10-5x-y$

$$\int_0^2\int_0^{10-5x}\int_0^{10-5x-y} \ln(x+y+z) \, dz \, dy \, dx$$