MA 242 Test 3 Version 2

1. (10 points) Sketch the region of integration and change the order of integration $\int_0^9 \int_0^{y/9} f(x,y)\,dx\,dy$

2. (20 points) **Set up** the integral needed to find the mass of the solid F bounded by the paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$ with density $\sigma(x,y,z) = e^{xz}$ using cylindrical coordinates. **Do not evaluate.**

3. (18 points) Find the average value of f(x,y)= y$\sqrt{x^2+y^2}$ over the lamina bounded by $x^2 + y^2 = 4$, where y ≥ 0

4. (22 points) a) Evaluate $\iiint_F z\,dV$, where F is the solid that lies above the xy-plane, under the hemisphere z=$\sqrt{4 - x^2 - y^2}$, and below the cone z=$\sqrt{x^2+y^2}$

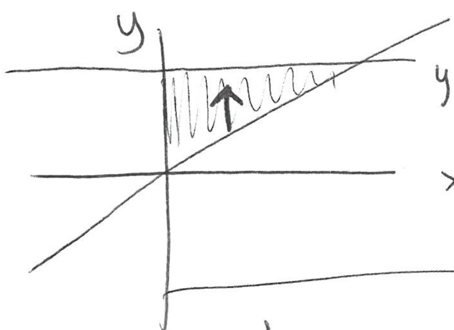
   b) **In general**, what is y in spherical coordinates?

5. (15 points) Use a double integral to find the volume of the solid bounded by the cylinder $z = 11 - x^2$, the planes $z = 10$, y=3, and the xz-plane.

6. (15 points) **Set up** the iterated integral $\iiint_F ln(x + y + z)\,dV$ if F is the solid tetrahedron with vertices (0,0,0), (2,0,0), (0,12,0), (0,0,12). **Do not evaluate.**

C3 T3V2 Solutions

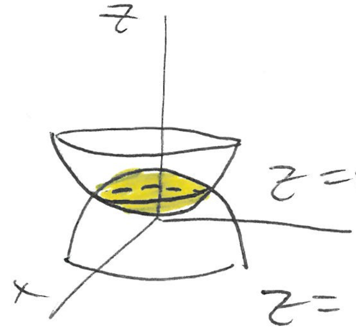
1. (10 points)



$x = y/9 \longleftrightarrow y = 9x$

$$\boxed{\int_0^1 \int_{9x}^{9} f(x,y)\, dy\, dx}$$

2. (20 points)



$z = x^2 + y^2$

$z = 18 - x^2 - y^2$

$$M = \iiint_F \sigma \, dV$$

$$= \iiint_F e^{xz} \, dV$$

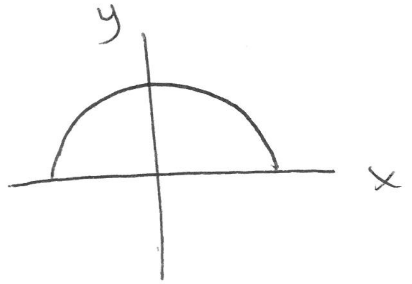
$$= \int_0^{2\pi} \int_0^3 \int_{r^2}^{18-r^2} e^{r\cos\theta\, z} \, dz\, r\, dr\, d\theta$$

$18 - r^2 = r^2$

$18 = 2r^2$

$9 = r^2$

3. (18 points)



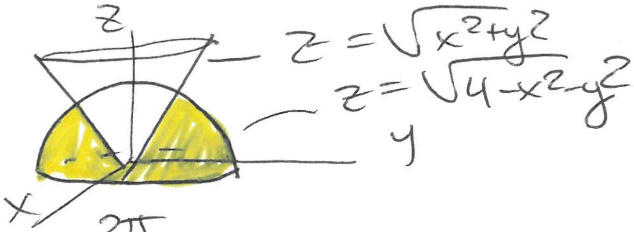
$$\frac{\iint_D f\,dA}{\iint_D 1\,dA}$$

$$= \frac{\iint y\sqrt{x^2+y^2}\,dA}{\iint_D 1\,dA} = \frac{\int_0^{\pi}\int_0^{2} r\sin\theta\sqrt{r^2}\,r\,dr\,d\theta}{\frac{1}{2}\pi(2)^2}$$

$$\frac{\int_0^{\pi}\sin\theta\,d\theta\int_0^{2} r^3\,dr}{2\pi} = \frac{-\cos\theta\Big|_0^{\pi}\ \frac{1}{4}r^4\Big|_0^{2}}{2\pi}$$

$$= \frac{(-\cos\pi+\cos 0)\,\frac{1}{4}\cdot 2^4}{2\pi} = \boxed{\frac{2\cdot\frac{1}{4}\cdot 2^4}{2\pi}} = \frac{4}{\pi}$$

4. (22 points)



$$\underline{\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^2 \rho\cos\phi\; \rho^2\sin\phi\, d\rho\, d\phi\, d\theta}$$

$$2\pi \int_{\pi/4}^{\pi/2} \cos\phi\sin\phi\, d\phi \quad \int_0^2 \rho^3\, d\rho$$

$u=\sin\phi$

$du=\cos\phi$

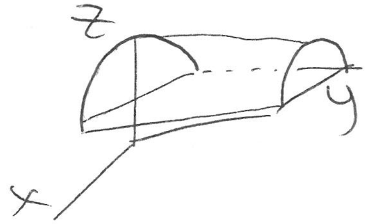
$$2\pi \int_{\sqrt{2}/2}^{1} u\,du \quad \frac{1}{4}\rho^4\Big|_0^2$$

$$\cancel{2}\pi\left(\cancel{\tfrac{1}{2}}u^2\right)\Big|_{\sqrt{2}/2}^{1} \quad \frac{1}{4}2^4$$

$$\boxed{\pi\left(1-\frac{1}{2}\right)\frac{1}{4}\cdot 2^4} \quad = 2\pi$$

b) $y=\rho\sin\phi\sin\theta$

5. (15 points)



$$-11-x^2=10$$
$$1=x^2$$

$$V=\int_{-1}^{1}\int_{0}^{3} 11-x^2-10\,dy\,dx$$

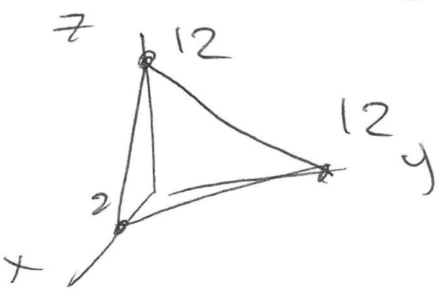
$$\int_{-1}^{1}\int_{0}^{3} 1-x^2\,dy\,dx = \int_{-1}^{1} (1-x^2)y\Big|_0^3\,dx$$

$$\int_{-1}^{1} \underbrace{3(1-x^2)}_{\text{even funct.}}dx = 2\left[3\left(x-\tfrac{1}{3}x^3\right)\right]_0^1$$

$$6\left(1-\tfrac{1}{3}\right) = 12/3 = 4$$

6. (15 points)

$$z=ax+by+c$$



$(0,0,12)$: $12=c$

$$z=12+ax+by$$

$(2,0,0)$: $0=12+2a$ $\quad a=-6$

$(0,12,0)$: $0=12+12b$ $\quad b=-1$

$$z=12-6x-y$$

$$\int_0^2\int_0^{12-6x}\int_0^{12-6x-y} \ln(x+y+z)\,dz\,dy\,dx$$