Name:____________________________ MA 341 Test 2 Version 1

1. (16 points) Use **the method of undetermined coefficients** to solve the Initial Value Problem (IVP): $y'' + 9y = 13e^{2t}$; y(0)=0, $y'(0)$=8

2. (14 points) Use **the method of variation of parameters** to find a particular solution to: $y'' - 2y' + y = \frac{e^t}{t^2+1}$

3. (12 points) A 64 lb weight attached to a spring stretches it 6 inches before coming to a rest at equilibrium. The damping constant is 4 lb - sec/ft. At time t = 0, the spring is compressed 3 inches and released. If y(t) is the position of the mass at time t, use 32 ft/s^2 for the gravitational constant and formulate the IVP that describes this system (**Do not solve it)**

4. (12 points) Use the definition of the Laplace transform to find the Laplace transform of f(t)=2e^{4t} and state its domain.

5. (7 points) Express the given function using unit step functions (do **not** find its Laplace transform) $f(t) = \begin{cases} 2t, & t < 6 \\ sin(4t), & 6 \le t \end{cases}$

Use the table below to answer the following:

$L\{y''\} = s^2L\{y\} - sy(0) - y'(0)$	$L\{\cos(bt)\} = \frac{s}{s^2 + b^2}$	$L\{e^{at}\} = \frac{1}{s - a}$
$L\{y'\} = sL\{y\} - y(0)$	$L\{\sin(bt)\} = \frac{b}{s^2 + b^2}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$
$L\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$	$L\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2 + b^2}$	$L\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2 + b^2}$
$L\{g(t)u(t-a)\} = e^{-as}L\{g(t+a)\}$	$L^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	$L\{t^n f(t)\} = (-1)^n \frac{d^n(F(s))}{ds^n}$
	$L\{1\} = \frac{1}{s}$	$L\{u(t-a)\} = \frac{e^{-as}}{s}$

6. (8 points) Find the inverse Laplace of the following: $\frac{e^{-7s}}{s^3}$

7. (15 points) Find the inverse Laplace of the following: $\frac{20+4s^2}{s(s^2-2s+10)}$

8. (16 points) Use the method of Laplace transforms to solve the Initial Value Problem: $y'' + 2y' = 6$; y(0)=0, $y'(0) = 1$

MA 341 T2 Solutions

1. (16 points) $r^2+9=0$ $\quad r^2=-9$ $\quad r=\pm 3i$

$$y_c = c_1\cos 3t + c_2\sin 3t$$

$$y_p = Ae^{2t} \quad y_p' = 2Ae^{2t} \quad y_p'' = 4Ae^{2t}$$

$$y = y_c + y_p \qquad y'' + 9y = 13e^{2t}$$

$$4Ae^{2t} + 9Ae^{2t} = 13e^{2t}$$

$$13A = 13 \quad A = 1$$

$$y = c_1\cos 3t + c_2\sin 3t + 1e^{2t}$$

$$y(0) = 0 = c_1 + \overset{1}{\cancel{e^0}} \qquad c_1 = -1$$

$$y = -\cos 3t + c_2\sin 3t + e^{2t}$$

$$y' = 3\sin 3t + 3c_2\cos 3t + 2e^{2t}$$

$$y'(0) = 8 = 3c_2 + 2$$

$$6 = 3c_2 \qquad c_2 = 2$$

$$\boxed{y = -\cos 3t + 2\sin 3t + e^{2t}}$$

2. (14 points)

$$r^2 - 2r + 1 = 0$$

$$(r-1)^2 = 0$$

$$y_c = c_1 e^t + c_2 t e^t$$

$$y_p = v_1 e^t + v_2 t e^t$$

$$v_1' = -v_2' t$$

$$-\left(v_1' e^t + v_2' t e^t = 0 \right)$$

$$v_1' e^t + v_2' (1e^t + te^t) = \frac{e^t}{t^2+1}$$

$$v_2' e^t = \frac{e^t}{t^2+1}$$

$$v_2' = \frac{1}{t^2+1}$$

$$v_2 = \tan^{-1} t$$

$$v_1' = -v_2' t$$

$$v_1' = -\frac{t}{t^2+1}$$

$$v_1 = \int \frac{-t}{t^2+1}\,dt \qquad u = t^2+1$$

$$du = 2t\,dt$$

$$= \int -\frac{1}{2}\frac{1}{u}\,du \qquad \frac{1}{2}du = t\,dt$$

$$= -\frac{1}{2}\ln(t^2+1)$$

$$\boxed{y_p = -\frac{1}{2}\ln(t^2+1)\, e^t + (\tan^{-1} t)\, t e^t}$$

3. (12 points)

$F = mg$

$64 = m\,32 \quad m = 2$

$F = ky$

$64 = k\left(\frac{1}{2}\right)$

$k = 128$

$my'' + by' + ky = 0$

$$\boxed{2y'' + 4y' + 128y = 0 \qquad y(0) = -\frac{1}{4},\ y'(0) = 0}$$

4. (12 points)

$$\mathcal{L}\{f\} = \int_0^\infty e^{-st} f\,dt$$

$$= \int_0^\infty e^{-st}\, 2e^{4t}\,dt$$

$$= \lim_{n\to\infty} \int_0^n 2e^{(4-s)t}\,dt$$

$$= \lim_{n\to\infty} \frac{2}{4-s} e^{(4-s)t}\Big|_0^n$$

$$= \lim_{n\to\infty} \frac{2}{4-s}\left[e^{(4-s)n} - \cancel{e^0}\,1\right] = \frac{2}{4-s}\left[0 - 1\right] = \boxed{\frac{-2}{4-s}}$$

$\boxed{s > 4}$

domain

5. (7 points)

$$\mathcal{L}\{f\} = 2t + (\sin 4t - 2t)\,u(t-6)$$

6. (8 points) $\mathcal{L}^{-1}\{\frac{1}{s^3}\} = \frac{1}{2}t^2$

$$\mathcal{L}^{-1}\{e^{-7s} f\} = u(t-7)\,f(t-7)$$
$$= u(t-7)\left[\tfrac{1}{2}(t-7)^2\right]$$

7. (15 points) $s^2 - 2s + 10 = (s-\alpha)^2 + \beta^2$

$= s^2 - 2\alpha s + \alpha^2 + \beta^2$

$\alpha = 1$

$\alpha^2 + \beta^2 = 10$

$\rightarrow \beta = 3$

$$\frac{20+4s^2}{s(s^2-2s+10)} = \frac{A}{s} + \frac{B(s-1)+3C}{(s-1)^2+3^2}$$

$As^2 - 2As + 10A + s(B(s-1) + 3C) = 20 + 4s^2$

$\underbrace{As^2} - \underline{2As} + 10A + \underbrace{Bs^2} - \underline{Bs} + 3\underline{Cs} = 20 + 4s^2$

$A + B = 4$

$-2A - B + 3C = 0$

$10A = 20$ $\quad A = 2,\ B = 2$ $\quad -4 - 2 + 3C = 0$

$C = 2$

$$\frac{2}{s} + \frac{2(s-1) + 2(3)}{(s-1)^2 + 3^2} \rightarrow \boxed{2 + 2e^{t}\cos 3t + 2e^{t}\sin 3t}$$

8. (16 points)

$$s^2L - s\cancel{y(0)} - \cancel{y'(0)} + 2[sL - \cancel{y(0)}] = \frac{6}{s}$$

$$(s^2+2s)L = \frac{6}{s} + 1$$

$$L = \frac{\frac{6}{s}+1}{s^2+2s} \cdot \frac{s}{s} = \frac{6+s}{s^3+2s^2} =$$

$$= \frac{6+s}{s^2(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

$$As(s+2) + B(s+2) + Cs^2 = 6+s$$

$$As^2 + 2As + Bs + 2B + Cs^2 = 6+s$$

$A+C=0$

$2A+B=1$ $\quad A=-1$ $\quad C=1$

$2B=6$ $\quad B=3$

$$\mathcal{L}\{f\} = \frac{-1}{s} + \frac{3}{s^2} + \frac{1}{s+2}$$

$$\boxed{y = -1 + 3t + e^{-2t}}$$