1. (26 points) Use $\vec{\mathbf{F}}$(x,y,z)=$\langle \tan^{-1} y,\ \frac{x}{1+y^2} + e^{4z} + 4,\ 4ye^{4z} + 4z \rangle$ to answer the following:
   a) Assume $\vec{\mathbf{F}}$(x,y,z) is conservative and find its most general potential function f
   b) Use the Fundamental theorem for Line Integrals to find the work done by $\vec{\mathbf{F}}$(x,y,z) moving a particle along the curve given by $\vec{\mathbf{r}}$(t)=$\langle t, 2 - t, \sqrt{2t} \rangle$ where 0$\leq t \leq 2$

2. (20 points) Use $\vec{\mathbf{F}}$(x,y,z)=$\langle x^2, \mathrm{xy}, \mathrm{z} - \mathrm{y} \rangle$ to answer the following:
   a) Show $\vec{\mathbf{F}}$(x,y,z)=$\langle x^2, \mathrm{xy}, \mathrm{z} - \mathrm{y} \rangle$ is not path independent
   b) Find the work done by $\vec{\mathbf{F}}$(x,y,z) moving a particle along the line segment from (1,0,1) to (1,3,5)

3. (15 points) Find the mass of the wire with density $\sigma(x, y, z) = \frac{z\sqrt{y}}{\sqrt{1+4x^2}}$ in the shape of the curve y=x^2 in the plane z=3 from (0,0,3) to (2,4,3)

4. (13 points) Use Green's theorem to find the circulation of the vector field $\vec{\mathbf{F}}$(x,y)=($\sqrt{x}$ +2y)**i** +(5x+sec^2 y)**j** moving clockwise along the triangle with vertices (0,0), (0,4), and (5,0).
   Hint: $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$

5. (16 points) Find the surface area of the portion of the paraboloid $z = 2x^2 + 2y^2$ where 0$\leq z \leq 2$

6. (10 points) Find the gradient vector field of f(x,y)=$y^2 - x^2$. Describe the direction of vectors in the first quadrant.

C3 T4 V1 Solutions

1. (26 points)

a) $f_x = \tan^{-1} y \quad f_y = \frac{x}{1+y^2} + e^{4z} + 4 \quad f_z = 4ye^{4z} + 4z$

$f = x\tan^{-1} y + g(y,z)$

$f_y = \frac{x}{1+y^2} + g_y = \frac{x}{1+y^2} + e^{4z} + 4$

$g_y = e^{4z} + 4$

$g = ye^{4z} + 4y + h(z)$

$f = x\tan^{-1} y + ye^{4z} + 4y + h(z)$

$f_z = 0 + 4ye^{4z} + 0 + h'(z) =$

$h'(z) = 4z$

$h(z) = 2z^2 + K$

$$\boxed{f = x\tan^{-1} y + ye^{4z} + 4y + 2z^2 + K}$$

b) $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

$\vec{r}(0) = \langle 0,2,0 \rangle \quad \vec{r}(2) = \langle 2,0,2 \rangle$

$f(2,0,2) - f(0,2,0) = 2\cancel{\tan}\,0 + \cancel{0}e^8 + 0 + 8 - [0 + 2e^0 + 8]$

$= \boxed{-2}$

2. (20 points)

a) $\text{curl}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xy & z-y \end{vmatrix}$

$= \langle -1-0, -(0-0), y-0 \rangle \neq \vec{0}$

b) $\vec{r}(t) = (1-t)\vec{r}_0 + \vec{r}_1 t$

$= (1-t)\langle 1,0,1\rangle + \langle 1,3,5\rangle t$

$= \langle 1-t, 0, 1-t\rangle + \langle t, 3t, 5t\rangle$

$= \langle 1, 3t, 1+4t\rangle$

$W = \int_C \langle x^2, xy, z-y\rangle \cdot d\vec{r}$

$= \int_0^1 \langle 1^2, 3t, \underbrace{1+4t-3t}_{1+t}\rangle \cdot \langle 0, 3, 4\rangle$

$= \int_0^1 9t + 4 + 4t \; dt$

$\int_0^1 13t + 4 \, dt = \frac{13}{2}t^2 + 4t \Big|_0^1$

$\boxed{\frac{13}{2} + 4}$

3. (15 points)

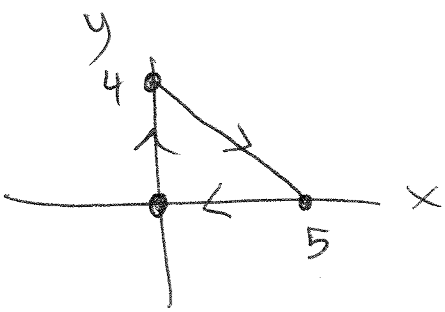
$$\left.\begin{array}{l} x=t \\ y=t^2 \\ z=3 \end{array}\right\} \vec{r}(t)=\langle t, t^2, 3\rangle \qquad 0\le t\le 2$$

$$m=\int_C \frac{z\sqrt{y}}{\sqrt{1+4x^2}}\,ds = \int_0^2 \frac{3\sqrt{t^2}}{\sqrt{1+4t^2}}\sqrt{1^2+(2t)^2+0^2}\,dt$$

$$\vec{r}\,'=\langle 1, 2t, 0\rangle$$

$$=\int_0^2 3t\,dt = \frac{3}{2}t^2\Big|_0^2 = \boxed{6}$$

4. (13 points)



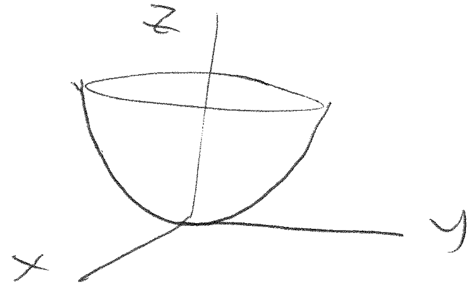
$$-\iint_D 5-2\,dA$$

$$=-\iint_D 3\,dA$$

$$=-3\left(\tfrac{1}{2}\cdot 5\cdot 4\right)$$

$$=\boxed{-30}$$

5. (16 points)



polar!

$2 = 2r^2$

$$S.A. = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

$$= \iint_D \sqrt{(4x)^2 + (4y)^2 + 1}\, dA$$

$$= \iint_D \sqrt{16x^2 + 16y^2 + 1}\, dA$$

$$\int_0^{2\pi} \int_0^1 \sqrt{16r^2+1}\; r\, dr\, d\theta$$

$u = 16r^2 + 1$

$du = 32r\, dr$

$\frac{1}{32} du = r\, dr$

$$2\pi \; \frac{1}{32} \int_1^{17} \sqrt{u}\, du$$

$$\frac{\pi}{16} \; \frac{2}{3} \; u^{3/2} \Big|_1^{17}$$

$$\boxed{\frac{\pi}{16}\left(\frac{2}{3}\right) \left[17^{3/2} - 1\right]}$$

6. (10 points)

$\nabla f = \langle -2x, 2y \rangle$ up & to the left