1. (26 points) Use $\vec{F}(x,y,z)=\langle \tan^{-1} y,\ \frac{x}{1+y^2} + e^{3z} + 3,\ 3ye^{3z} + 3\rangle$ to answer the following:
   a) Assume $\vec{F}(x,y,z)$ is conservative and find its most general potential function f
   b) Use the Fundamental theorem for Line Integrals to find the work done by $\vec{F}(x,y,z)$ moving a particle along the curve given by $\vec{r}(t)=\langle t, 3-t, \sqrt{3t}\rangle$ where $0\le t \le 3$

2. (20 points) Use $\vec{F}(x,y,z)=\langle x^2, xy, z-y\rangle$ to answer the following:
   a) Show $\vec{F}(x,y,z)=\langle x^2, xy, z-y\rangle$ is not path independent
   b) Find the work done by $\vec{F}(x,y,z)$ moving a particle along the line segment from (1,0,1) to (1,2,4)

3. (15 points) Find the mass of the wire with density $\sigma(x,y,z) = \frac{z\sqrt{y}}{\sqrt{1+4x^2}}$ in the shape of the curve y=x^2 in the plane z=5 from (0,0,5) to (2,4,5)

4. (13 points) Use Green's theorem to find the circulation of the vector field $\vec{\mathbf{F}}$(x,y)=($\sqrt{x}$ +2y)**i** +(6x+sec^2 y)**j** moving clockwise along the triangle with vertices (0,0), (0,3), and (4,0).
   Hint: $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$

5. (16 points) Find the surface area of the portion of the paraboloid $z = 2x^2 + 2y^2$ where $0\le z \le 2$

6. (10 points) Find the gradient vector field of f(x,y)=$x^2 - y^2$. Describe the direction of vectors in the first quadrant.

C3 T4 V2 Solutions

1. (26 points)

a) $f_x = \tan^{-1} y \qquad f_y = \frac{x}{1+y^2} + e^{3z} + 3, \quad f_z = 3ye^{3z} + 3$

$f = x\tan^{-1}y + g(y,z)$

$f_y = \frac{x}{1+y^2} + g_y =$

$g_y = e^{3z} + 3$

$g = ye^{3z} + 3y + h(z)$

$f = x\tan^{-1}y + ye^{3z} + 3y + h(z)$

$f_z = 0 + 3ye^{3z} + 0 + h'(z) =$

$h'(z) = 3$

$h(z) = 3z + k$

$$\boxed{f = x\tan^{-1}y + ye^{3z} + 3y + 3z + k}$$

b) $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

$= f(\vec{r}(3)) - f(\vec{r}(0)) = f(3,0,3) - f(0,3,0)$

$= 3\cancel{\tan^{-1}}0 + \cancel{0}e^9 + 0 + 9 - [\cancel{0\tan^{-1}}3 + 3e^0 + 9 + 0]$

$= \boxed{-3}$

2. (20 points)

a) $\text{curl}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xy & z-y \end{vmatrix}$

$= \langle -1-0, -(0-0), y-0 \rangle \neq \vec{0}$ ✓

b) $\vec{r}(t) = (1-t)\vec{r_0} + \vec{r_1}t$

$= (1-t)\langle 1, 0, 1 \rangle + \langle 1, 2, 4 \rangle t$

$= \langle 1-t, 0, 1-t \rangle + \langle t, 2t, 4t \rangle$

$= \langle 1, 2t, 1+3t \rangle$

$W = \int_C \langle x^2, xy, z-y \rangle \cdot d\vec{r}$

$= \int_0^1 \langle 1^2, 1\cdot 2t, \underbrace{1+3t-2t}_{1+t} \rangle \cdot \langle 0, 2, 3 \rangle\, dt$

$= \int_0^1 0 + 4t + 3 + 3t \; dt$

$\int_0^1 3 + 7t \; dt = 3t + \frac{7}{2}t^2 \Big|_0^1$

$\boxed{3 + \frac{7}{2}}$

3. (15 points)

$x=t$

$y=t^2$

$z=5$

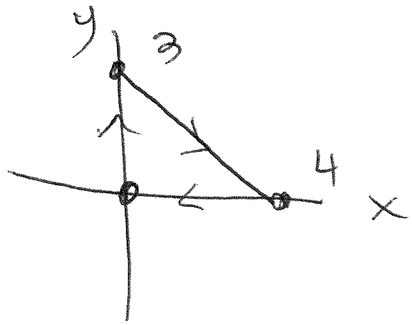
$\vec{r}=\langle t, t^2, 5\rangle$

$\vec{r}\,'=\langle 1, 2t, 0\rangle$

$$m=\int_C \frac{z\sqrt{y}}{\sqrt{1+4x^2}}\,ds = \int_0^2 \frac{5\sqrt{t^2}}{\cancel{\sqrt{1+4t^2}}}\cancel{\sqrt{1^2+(2t)^2+0^2}}\,dt$$

$$\int_0^2 5t\,dt = \frac{5}{2}t^2\Big|_0^2 = \boxed{10}$$

4. (13 points)



$$-\iint_D 6-2\,dA$$

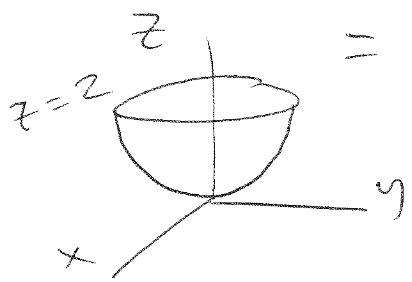
$$-\iint_D 4\,dA$$

$$= -4\left(\tfrac{1}{2}4\cdot 3\right)$$

$$= \boxed{-24}$$

5. (16 points)

$$S.A. = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$



$$= \iint_D \sqrt{(4x)^2 + (4y)^2 + 1}\, dA$$

$$\iint_D \sqrt{16x^2 + 16y^2 + 1}\, dA$$

$$\iint_D \sqrt{16r^2 + 1}\; r\, dr\, d\theta$$

$2 = 2r^2$

$r = 1$

$$\int_0^{2\pi} \int_0^1 \sqrt{16r^2 + 1}\; r\, dr\, d\theta$$

$$2\pi \, \frac{1}{32} \int_1^{17} \sqrt{u}\, du$$

$u = 16r^2 + 1$

$du = 32r\, dr$

$\frac{1}{32}\, du = r\, dr$

$$\frac{\pi}{16} \, \frac{2}{3} u^{3/2} \Big|_1^{17}$$

$$\boxed{\frac{\pi}{16}\left(\frac{2}{3}\right)\left[17^{3/2} - 1\right]}$$

6. (10 points)

$\nabla f = \langle 2x, -2y \rangle$

down & to the right.