MA 241 Test 1 Version 1

You may need the following on your test:

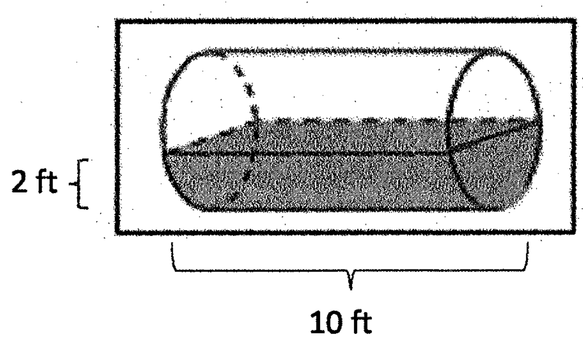
density of water $= 1000\ kg/m^3$
weight density of water $= 62.4\ lb/ft^3$
gravity $= 9.8\ m/s^2$
gravity $= 32\ ft/s^2$

1. (12 points) Find the average value of f(x)=$\frac{x}{x^2+1}$ from $0 \le x \le 3$

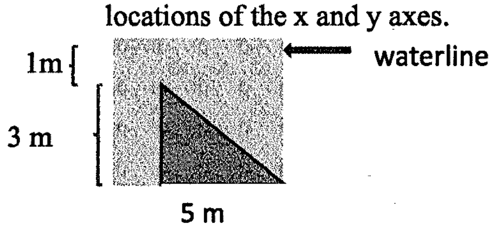
2. (14 points) A spring has a natural length of 10 inches. A force of 5 lb is required to keep it stretched to a length of 12 inches. Find the work needed to stretch the spring from 10 inches to 14 inches. Include units with your answer.

3. (16 points) A lamina in the first quadrant is bounded by y=e^x, the x-axis, the y-axis, and x=1. Find its centroid.

4. (16 points) A cylindrical tank is half filled with water as shown below. Set up (**do not** evaluate) the integral needed to find the work required to pump all the water out of the top of the tank. Your answer should include units and a picture with locations of the x and y axes.



5. (15 points) Set up the integral needed to find the hydrostatic force on the submerged vertical plate pictured below. Your answer should include units and a picture with locations of the x and y axes.



6. (13 points) A 15 ft long chain weighs 30 pounds. Find the work to raise one end to a height of 10 ft. Include units with your answer.

7. (14 points) Find the length of the curve given by $y = \frac{(x^2+2)^{3/2}}{3}, 0 \le x \le 1$

C2 T1 V1 Solutions

1. (12 points)

$$f_{ave} = \frac{1}{b-a}\int_a^b f\,dx$$

$$= \frac{1}{3-0}\int_0^3 \frac{x}{x^2+1}\,dx \qquad \begin{aligned} u &= x^2+1 \\ du &= 2x\,dx \\ \tfrac{1}{2}\,du &= x\,dx \end{aligned}$$

$$= \frac{1}{3}\int_1^{10} \frac{1}{2}\frac{1}{u}\,du$$

$$\frac{1}{6}\ln u\Big|_1^{10} = \frac{1}{6}(\ln 10 - \ln 1)$$

$$\boxed{\frac{1}{6}\ln 10}$$

2. (14 points)

$$F = kx$$

$$5 = k\left(\frac{2}{12}\right)$$

$$k = 30$$

$$W = \int_a^b kx\,dx$$

$$= \int_0^{\frac{1}{3}} 30x\,dx = 15x^2\Big|_0^{1/3} = \boxed{15\left(\tfrac{1}{9}\right)\ \text{ft-lbs}}$$

3. (16 points)

$$\bar{x} = \frac{\int_0^1 x e^x \, dx}{\int_0^1 e^x \, dx} = \frac{xe^x\big|_0^1 - \int_0^1 e^x \, dx}{e^x\big|_0^1} = \frac{xe^x - e^x\big|_0^1}{e-1}$$

$\int_0^1 x e^x \, dx$ LIATE

$u = x \qquad v = e^x$

$du = dx \qquad dv = e^x \, dx$

$uv - \int v \, du$

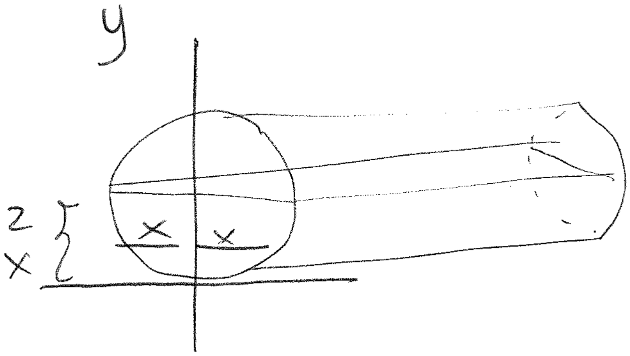
$$\bar{x} = \frac{e - e - (0 - e^0)}{e-1} = \frac{1}{e-1}$$

$$\bar{y} = \frac{\int_0^1 \frac{1}{2}(e^x)^2 \, dx}{e-1} = \frac{\int_0^1 \frac{1}{2} e^{2x} \, dx}{e-1} = \frac{\frac{1}{4} e^{2x}\big|_0^1}{e-1}$$

$$= \frac{\frac{1}{4}e^2 - \frac{1}{4}}{e-1}$$

$(\bar{x}, \bar{y})$

4. (16 points)



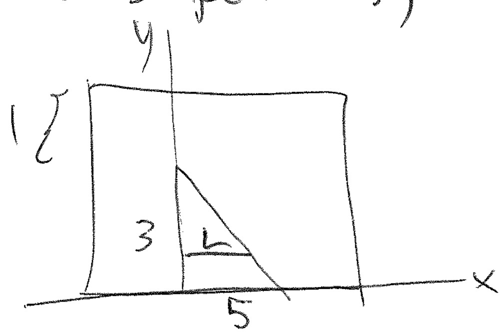
$$(x-0)^2+(y-2)^2=2^2$$

$$x=\sqrt{4-(y-2)^2}$$

$$W=\int_a^b \rho\, A(y)\,(h-y)\,dy$$

$$=\int_0^2 62.4\left[2\sqrt{4-(y-2)^2}\right](10)(4-y)\,dy \quad \text{ft-lb}$$

5. (15 points)



$$\frac{L}{3-y}=\frac{5}{3}$$

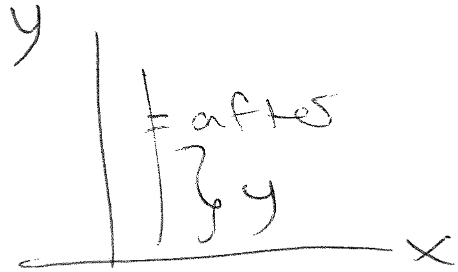
$$F=\int_a^b \rho\, L(y)\,(\text{depth})\,dy$$

$$\boxed{F=\int_0^3 1000(9.8)\left[\tfrac{5}{3}(3-y)\right](4-y)\,dy \quad \text{N}}$$

6. (13 points).

$\frac{30}{15} = 2\,\text{lb/ft}$

Before



$W_{piece} \approx Fd = 2(\Delta y)y$

$$W = \int_0^{10} 2y\,dy = y^2 \Big|_0^{10} = 100 \text{ ft-lbs}$$

7. (14 points)

$$y' = \frac{3}{2}\,\frac{1}{3}(x^2+2)^{1/2}\,2x = x\sqrt{x^2+2}$$

$$L = \int_0^1 \sqrt{(x\sqrt{x^2+2})^2 + 1}\;dx$$

$$= \int_0^1 \sqrt{x^2(x^2+2) + 1}\;dx$$

$$= \int_0^1 \sqrt{x^4+2x^2+1}\;dx$$

$$\int_0^1 \sqrt{(x^2+1)^2}\;dx$$

$$= \int_0^1 x^2+1\,dx = \frac{1}{3}x^3 + x\Big|_0^1 = \boxed{\frac{1}{3}+1}$$