Name: ________________________ MA 341 Test 1 Version 1

1. (20 points) Use the Initial Value Problem (IVP): $(x^2 - 1)\frac{dy}{dx} = xy\,,\ y(\sqrt{2}) = 3$ to answer the following:
   a) Is it separable, linear, or both? No explanation is required.
   b) Solve the IVP. Write your answer with y as an explicit function of x if possible.
   c) Does the Existence and Uniqueness theorem guarantee that this is a unique solution? Justify your answer

2. (16 points) Solve the IVP $\frac{dy}{dx} - 2y = 6xe^{2x}$, $y(1) = 0$ Write your answer with y as an explicit function of x if possible.

3. (12 points) A large tank initially contains 100 liters of brine in which 7 kg of salt has been dissolved. Brine solution flows into the tank at a rate of 6 L/min. The well-mixed solution leaves the tank at a rate of 8 L/min. If the concentration of salt in the brine entering the tank is 0.1 kg/L and x(t) is the amount of salt in the tank at time t, formulate the IVP that describes this system. **Do not solve the differential equation.**

4. (13 points) Use $(2\cos(2x) + y)dx + \left(x + \frac{1}{1+y^2}\right)dy = 0$ to answer the following:

a) Show that this is an exact differential equation
b) Find its implicit general solution

5. (14 points) Solve the Initial Value Problem (IVP):
   $y'' - 6y' + 10y = 0, \; y(0) = 2, \; y'(0) = 0$

6. (12 points) Use the differential equation $\frac{dy}{dt} = y^2(4 - y)$ to answer the following:

   a) Sketch its phase line and classify its equilibria as we have done in class

   b) Use the phase line to determine the asymptotic behavior as $t \to \infty$ of the solution through $y(0)=1$

   c) If $y(0)=4$, without solving the differential equation, find $y(11)$

7. (13 points) Determine if $xy^{-1} + y = 1$ is a solution to $\frac{dy}{dx} = \frac{y}{x - y^2}$

MA 341 T1 Solutions

1. (20 points)

$$\frac{dy}{dx} = \frac{xy}{x^2-1}$$

a) both

b) $\int \frac{dy}{y} = \int \frac{x}{x^2-1}\,dx$ $\quad u = x^2-1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\ln|y| = \int \frac{1}{2}\frac{1}{u}\,du$

$\ln|y| = \frac{1}{2}\ln|u| + C$

$\ln|y| = \frac{1}{2}\ln|x^2-1| + C$

$y = K\sqrt{x^2-1}$

$y(\sqrt{2}) = 3 = K\sqrt{2-1}$

$$\boxed{y = 3\sqrt{x^2-1}}$$

c) $f = \frac{xy}{x^2-1}$ $\quad$ f is cont at & around $(\sqrt{2}, 3)$

$\frac{\partial f}{\partial y} = \frac{x}{x^2-1}$ $\quad$ $\frac{\partial f}{\partial y}$ is cont at & around $(\sqrt{2}, 3)$

Yes

2. (16 points)

$$\frac{dy}{dx} - 2y = 6xe^{2x}$$

$$\mu = e^{\int -2\,dx} = e^{-2x}$$

$$\frac{d}{dx}\left[e^{-2x}y\right] = 6xe^{2x}e^{-2x} = 6x$$

$$e^{-2x}y = \int 6x\,dx$$

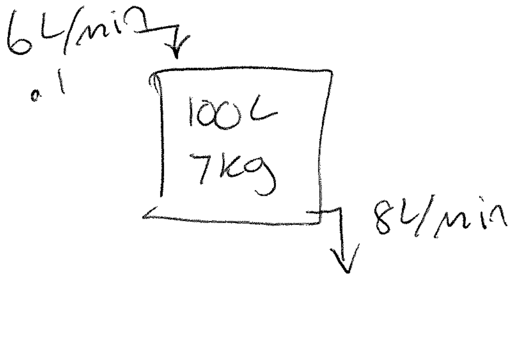
$$= 3x^2 + C$$

$$y = e^{2x}(3x^2 + C)$$

$$y(1) = 0 = e^2(3 + C)$$

$$\boxed{y = e^{2x}(3x^2 - 3)}$$

3. (12 points)



$$\frac{dx}{dt} = F_iC_i - F_oC_o$$

$$\frac{dx}{dt} = 6(.1) - 8\left(\frac{x}{100-2t}\right)$$

$$x(0) = 7$$

4. (13 points)

a) $M_y = 1 = N_x = 1$ ✓

b) $F_x = 2\cos 2x + y$

$F = \sin 2x + xy + g(y)$

$F_y = x + g'(y) = x + \frac{1}{1+y^2}$

$g(y) = \tan^{-1} y$

$\boxed{\sin 2x + xy + \tan^{-1} y = C}$

5. (14 points)

$r^2 - 6r + 10 = 0$

$r = \frac{6 \pm \sqrt{36-40}}{2} = \frac{6 \pm \sqrt{-4}}{2} = 3 \pm i$

$y = e^{3t}[c_1 \cos t + c_2 \sin t]$

$y(0) = 2 = e^0[c_1 \cos 0 + \cancel{c_2 \sin 0}]$ $c_1 = 2$

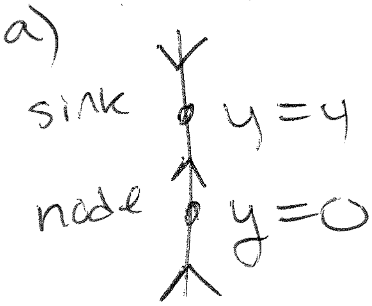
$y' = 3e^{3t}[2\cos t + c_2 \sin t] + e^{3t}[-2\sin t + c_2 \cos t]$

$y'(0) = 0 = 3[2] + c_2$ $c_2 = -6$

$\boxed{y = e^{3t}[2\cos t - 6\sin t]}$

6. (12 points)

a)



b) $y \to 4$

c) $y(11) = 4$

7. (13 points)

$$y^{-1} + x(-y^{-2})\frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$\left(\frac{-x}{y^2} + 1\right)\frac{dy}{dx} = -1/y$$

$$\frac{dy}{dx} = \frac{-1/y}{\frac{-x}{y^2}+1} \cdot \frac{y^2}{y^2} = \frac{-y}{-x+y^2} \cdot \frac{-1}{-1}$$

$$= \frac{y}{x-y^2} \checkmark$$

Yes