MA 241 Test 2 Version 1 Remember: No work=No credit!

1. (15 points) Evaluate $\int \frac{-2+3x^2}{x^2(x-1)}\,dx$

2. (14 points) Evaluate $\int_0^{\frac{\pi}{6}} \cos^3 x\,dx$

3. (15 points) Use **trig substitution** to evaluate $\int \frac{x}{\sqrt{9+x^2}}\,dx$

Hint: $x = 3\tan\theta$

4. (26 points) Determine if the following integrals are convergent or divergent. Evaluate the convergent integrals.

a) $\int_0^{\infty} \frac{dx}{x^2+25}$

b) $\int_1^9 \frac{dx}{\sqrt[3]{x-1}}$

5. (14 points) $\int \tan^3 x \sec x\,dx$

6. (16 points) Use $\int_1^9 \ln\sqrt{x}\,dx$ to answer the following:

a) Use Simpson's Rule to approximate the integral with n=4

b) Find the upper bound of the error estimate using $|E_S| \le \frac{K(b-a)^5}{180n^4}$ where $|f^{(4)}(x)| \le K$ for $a \le x \le b$. Hint: $f^{(4)}(x) = \frac{-3}{x^4}$

C2 T2 V1 Solutions

1. (15 points)

$$\int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}\,dx$$

$$Ax(x-1) + B(x-1) + Cx^2 = -2 + 3x^2$$

$$\underbrace{Ax^2} - \underline{Ax} + \underline{Bx} - B + \underbrace{Cx^2} =$$

$$A + C = 3$$

$$-A + B = 0$$

$$-B = -2 \quad B = 2,\ A = 2 \quad C = 1$$

$$\int \frac{2}{x} + \frac{2}{x^2} + \frac{1}{x-1}\,dx = \boxed{2\ln|x| - \frac{2}{x} + \ln|x-1| + C}$$

2. (14 points)

$$\int_0^{\pi/6} \cos^2 x \cos x\,dx = \int_0^{\pi/6} (1 - \sin^2 x)\cos x\,dx$$

$u = \sin x \quad du = \cos x\,dx$

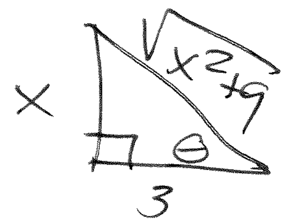
$u(0) = \sin 0 = 0 \quad u(\pi/6) = \sin \pi/6 = 1/2$

$$\int_0^{1/2} (1 - u^2)\,du = u - \frac{1}{3}u^3 \Big|_0^{1/2} = \boxed{\frac{1}{2} - \frac{1}{3}\left(\frac{1}{2}\right)^3}$$

3. (15 points)

$$x = 3\tan\theta$$
$$dx = 3\sec^2\theta\, d\theta$$

$$\frac{x}{3} = \tan\theta = \frac{opp}{adj}$$



$$\int \frac{3\tan\theta\, 3\sec^2\theta\, d\theta}{\sqrt{9+9\tan^2\theta}}$$

$$\int \frac{3\tan\theta\, 3\sec^2\theta\, d\theta}{\sqrt{9\sec^2\theta}}$$

$$\int \frac{3\tan\theta\, \cancel{3}\sec^{\cancel{2}}\theta}{\cancel{3}\cancel{\sec\theta}}\, d\theta$$

$$= 3\sec\theta + C$$

$$= 3\,\frac{hyp}{adj} + C$$

$$= \boxed{3\left(\frac{\sqrt{x^2+9}}{3}\right) + C}$$

4. (26)

a) $\int_0^\infty \frac{dx}{x^2+25} = \lim_{t\to\infty} \int_0^t \frac{dx}{x^2+25}$

$= \lim_{t\to\infty} \frac{1}{5}\tan^{-1}\frac{x}{5}\Big|_0^t$

$= \lim_{t\to\infty} \frac{1}{5}\left(\tan^{-1}\frac{t}{5} - \tan^{-1}0\right)$

$= \frac{1}{5}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{10}$ converges

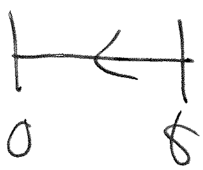
b) $\int_1^9 \frac{dx}{\sqrt[3]{x-1}}$ $\quad u = x-1 \quad du = dx$

$u(1) = 1-1 = 0$

$u(9) = 9-1 = 8$

$\int_0^8 \frac{du}{\sqrt[3]{u}}$

$\lim_{t\to 0^+} \int_t^8 u^{-1/3}\,du$



$= \lim_{t\to 0^+} \frac{3}{2}u^{2/3}\Big|_t^8$

$\lim_{t\to 0^+} \frac{3}{2}\left[8^{2/3} - t^{2/3}\right]$

$= \frac{3}{2}\left[8^{2/3}\right]$ conv

$= 6$

5. (14 points)

$$\int \tan^2 x \tan x \sec x \, dx$$

$$\int (\sec^2 x - 1) \tan x \sec x \, dx$$

$u = \sec x$

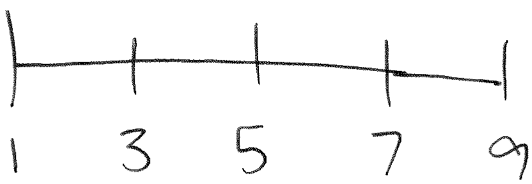
$du = \sec x \tan x \, dx$

$$\int (u^2 - 1) \, du = \frac{1}{3}u^3 - u + C$$

$$\boxed{\frac{1}{3}\sec^3 x - \sec x + C}$$

6. (16 points)

a) $\Delta x = \frac{b-a}{n} = \frac{9-1}{4} = 2$



$$S_4 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

$$= \frac{2}{3}\left[f(1) + 4f(3) + 2f(5) + 4f(7) + f(9)\right]$$

$$= \frac{2}{3}\left[\cancel{\ln 1}^{\,0} + 4\ln\sqrt{3} + 2\ln\sqrt{5} + 4\ln\sqrt{7} + \ln\sqrt{9}\right]$$

b) $$|E_S| \leq \boxed{\frac{\frac{3}{14}(9-1)^5}{180 \cdot 4^4}}$$