1. (15 points) Solve the Initial Value Problem (IVP). Write y as an explicit function of x, if possible.
   $\frac{dy}{dx} = \frac{\sec^2 x}{y}$; y(0)= -3

2. (17 points) Solve the IVP: $y'' + 2y' = -16\sin(2x)$; y(0)=0, $y'(0)$=0

3. (15 points) A 2 kg mass attached to the end of a hanging spring stretches the spring 2 m upon coming to rest at e2quilibrium. Its damping constant is 8 Ns/m. The mass is compressed 2 m from its equilibrium position and released. If x(t) is the position of the mass at time t, solve this initial value problem. What kind of damping is this? Hint: If it is needed, use g= 10 m/s^2

4. (13 points) Use $y'' - 6y' + 9y$ =f(x) **to answer the following:**
   a) Find its complementary solution, y_c
   b) Find the form of y_p if f(x)=9e^{3x}-2x^2. **Do not solve for coefficients.**

5. (13 points) Use Euler's method with a step size of 0.1 to estimate y(2.1) and y(2.2), if $y'=x(y+1)$; $y(2) = 0$. Clearly label your answers. **You don't need to simplify your estimate for y(2.2).**

6. (12 points) A large tank initially contains 300 L of brine in which 9 kg of salt has been dissolved. At time t=0, brine containing 0.2 kg/L enters the tank at a rate of 5 L/min. The well-mixed solution leaves the tank at rate at the same rate. If x(t) is the amount of salt in the tank at time t, formulate the IVP that describes this system. **Do not solve it.**

7. (15 points) Find the orthogonal trajectories of $e^y = kx$.

C2 T3 V1 Solutions

1. (15 points)

$$\int y\,dy = \int \sec^2 x\,dx$$

$$\frac{1}{2}y^2 = \tan x + C$$

$$y^2 = 2\tan x + C_1$$

$$y = \pm\sqrt{2\tan x + C_1}$$

$$y(0) = -3 = -\sqrt{2\cancel{\tan 0} + C_1}$$

$$\boxed{y = -\sqrt{2\tan x + 9}}$$

2. (17 points)

$$r^2 + 2r = 0$$

$$r(r+2) = 0$$

$$y_c = C_1 + C_2 e^{-2x}$$

$$y_p = A\cos 2x + B\sin 2x$$

$$y_p' = -2A\sin 2x + 2B\cos 2x$$

$$y_p'' = -4A\cos 2x - 4B\sin 2x$$

$$-4A\cos 2x - 4B\sin 2x + 2(-2A\sin 2x + 2B\cos 2x) = -16\sin 2x$$

$$-4A + 4B = 0 \quad A = B$$

$$-4B - 4A = -16$$

$$-8B = -16 \quad B = 2 \qquad y = C_1 + C_2 e^{-2x} + 2\cos 2x + 2\sin 2x$$

$$y(0) = 0 = C_1 + C_2 + 2 \qquad y' = 0 - 2C_2 e^{-2x} - 4\sin 2x + 4\cos 2x$$

$$y'(0) = 0 = -2C_2 + 4 = 0 \quad C_2 = 2 \quad C_1 = -4$$

$$\boxed{y = -4 + 2e^{-2x} + 2\cos 2x + 2\sin 2x}$$

3. (15 points)

$$mx'' + bx' + kx = F_{ext}$$

$$\boxed{2x'' + 8x' + 10x = 0 \quad x(0) = -2 \quad x'(0) = 0}$$

$F = kx$

$2(10) = k \cdot 2$

$$2r^2 + 8r + 10 = 0$$

$$r^2 + 4r + 5 = 0$$

$$r = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= -2 \pm 2i$$

$$x = e^{-2t}\left[c_1 \cos 2t + c_2 \sin 2t\right]$$

$$x(0) = c_1 = -2$$

$$x' = -2e^{-2t}\left[c_1 \cos 2t + c_2 \sin 2t\right] + e^{-2t}\left[-2c_1 \sin 2t + 2c_2 \cos 2t\right]$$

$$x'(0) = 0 = -2\left[c_1\right] + 1\left[2c_2\right]$$

$$0 = 4 + 2c_2 \qquad c_2 = -2$$

$$\boxed{x = e^{-2t}\left[-2\cos 2t - 2\sin 2t\right]}$$

$\boxed{\text{underdamping}}$

4. (13 points)

$$r^2 - 6r + 9 = 0$$
$$(r-3)^2 = 0$$

a) $y_c = C_1 e^{3x} + C_2 x e^{3x}$

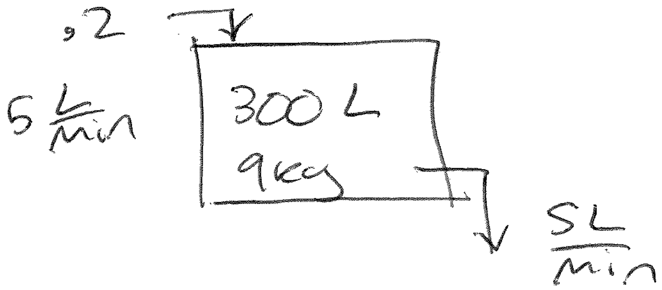
b) $y_p = Ae^{3x}x^2 + Bx^2 + Cx + D$

5. (13 points)

$$y(2.1) \approx y_1 = y_0 + f(x_0, y_0)\Delta x$$
$$= 0 + f(2,0)(.1)$$
$$= 2(0+1)(.1) = .2$$

$$y(2.2) \approx y_2 = y_1 + f(x_1, y_1)\Delta x$$
$$= .2 + f(2.2, .2)(.1)$$
$$= .2 + 2.2(.2+1)(.1)$$

b. (12 points)



$$\frac{dx}{dT} = F_i C_i - F_o C_o = 5(.2) - 5\left(\frac{x}{300}\right)$$

$$x(0) = 9$$

7. (15)

$$e^y = kx$$

$$e^y \frac{dy}{dx} = k$$

$$\frac{dy}{dx} = \frac{k}{e^y}$$

$$\perp: \frac{dy}{dx} = \frac{-e^y}{k}$$

$$k = \frac{e^y}{x} \quad \therefore \quad \frac{dy}{dx} = \frac{-e^y}{\frac{e^y}{x}} = -x$$

$$\int dy = \int -x \, dx$$

$$\boxed{y = -\frac{1}{2}x^2 + C}$$