MA 242 Test 2 Version 1

1. (17 points) Find and classify all critical points of f(x,y)= $x^4 - 4xy + 2y^2$. Justify your answers as we have done in class.

2. (20 points) Use the function f(x,y)=ln$(x^3 + 2y + 1)$ to find the following:
   a) Find the directional derivative of f(x,y) at P(1,1) in the direction point Q(2,3)
   b) Find the direction of the greatest rate of change of f(x,y) at P(1,1)

3. (20 points) Find the global maximum and minimum values of f(x,y)=$2x^2 - 4y^2$ over the region D bounded by the x-axis, $y = \sqrt{x}$, and x=4. Fully justify your answers as we have done in class.

4. (14 points) Use Lagrange multipliers to find the maximum and minimum values of the function f(x,y)= e^{2xy} subject to the given constraint, $x^2 + y^2 = 18$

5. (15 points) Use $\vec{r}(t)=\langle 3cos(t), 4cos(t), 5sin(t)\rangle$ to answer the following:
   a) Find the length of $\vec{r}(t)$ where $0 \leq t \leq 2\pi$
   b) Find a vector equation for the tangent line to $\vec{r}(t)$ when t=0

6. (14 points) The volume of a right circular cylinder with radius r and height h is V= $\pi r^2 h$. Find $\frac{dV}{dt}$ at the instant when r and h are both 2m if it is known that r is increasing at a rate of 2m/sec and h is increasing at a rate of 3m/sec.

C3T2V1 Solutions

1. (17 points)

$f_x = 4x^3 - 4y$ $\qquad 4x^3 - 4x = 4x(x^2-1) = 0$

$\qquad\qquad x=0,\ x=1,\ x=-1$

$f_y = -4x + 4y = 0 \quad y = x$

$f_{xx} = 12x^2$

$f_{yy} = 4$

$f_{xy} = -4$

$D = 48x^2 - 16$

$D(0,0) = -16 < 0$ saddle pt

$D(1,1) = 48 - 16 > 0$ $\cup$ local min

$f_{xx}(1,1) = 12 > 0$

$D(-1,-1) = 48 - 16 > 0$

~~$f_{xx}(-1,-1) = 48 > 0$~~

$f_{xx}(-1,-1) = 12 > 0$ $\cup$ local min

2. (20 points)

a) $\nabla f = \left\langle \frac{3x^2}{x^3+2y+1}, \frac{2}{x^3+2y+1} \right\rangle$

$\nabla f(1,1) = \left\langle \frac{3}{4}, \frac{2}{4} \right\rangle$

$\overrightarrow{PQ} = \langle 2-1, 3-1 \rangle = \langle 1, 2 \rangle$

$\hat{u} = \frac{\langle 1,2 \rangle}{\sqrt{5}} = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

$D_{\hat{u}} f = \left\langle \frac{3}{4}, \frac{2}{4} \right\rangle \cdot \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle = \boxed{\frac{3}{4\sqrt{5}} + \frac{4}{4\sqrt{5}}}$

b) $\left\langle \frac{3}{4}, \frac{2}{4} \right\rangle$

3. (20 points)

$f = 2x^2 - 4y^2$

Candidates

$f(0,0) = 0$

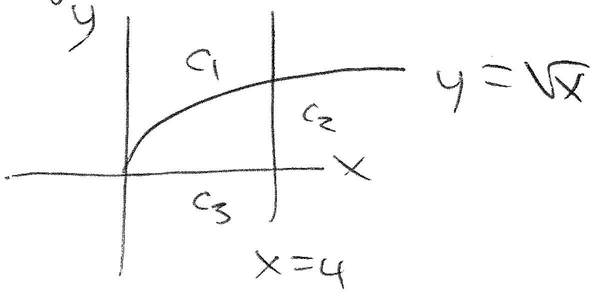
$f(1,1) = 2 - 4 = -2$

$f(4,0) = 32$

$f(0,0) = 0$

$f(4,2) = 32 - 16 = 16$

$\left.\begin{array}{l} f_x = 4x = 0 \\ f_y = -8y = 0 \end{array}\right\}$ critical pt $(0,0)$



C_1: $y = \sqrt{x}$ $\quad f(x, \sqrt{x}) = 2x^2 - 4x$

$f'(x, \sqrt{x}) = 4x - 4 = 0$

$x = 1$

C_2: $x = 4$ $\quad f(4,y) = 2(16) - 4y^2$

$f'(4,y) = -8y = 0 \quad y = 0$

C_3: $y = 0$ $\quad f(x,0) = 2x^2$

$f'(x,0) = 4x = 0 \quad x = 0$

Global max value $= 32$

Global min value $= -2$

4. (14 points)

$$\nabla f = \lambda \nabla g$$

$$\langle 2ye^{2xy}, 2xe^{2xy} \rangle = \lambda \langle 2x, 2y \rangle$$

$2ye^{2xy} = \lambda 2x \xrightarrow{\text{mult by } x} 2xye^{2xy} = 2\lambda x^2$

$2xe^{2xy} = \lambda 2y \xrightarrow{\text{mult by } y} 2xye^{2xy} = 2\lambda y^2$

$$x^2 = y^2$$

$$x = \pm y$$

$x^2 = y^2$: $\quad x^2 + x^2 = 18$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

$x = 3$ — $y = 3$, $y = -3$

$x = -3$ — $y = 3$, $y = -3$

$f(3,3) = e^{2\cdot 3\cdot 3} = e^{18}$ ← max

$f(-3,3) = e^{-18}$

$f(3,-3) = e^{-18}$ ← min

$f(-3,-3) = e^{18}$

5. (15 pts)

a) $\vec{r}' = \langle -3\sin t, -4\sin t, 5\cos t \rangle$

$$L = \int_0^{2\pi} \|\vec{r}'(t)\| \, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 16\sin^2 t + 25\cos^2 t} \; dt$$

$$\int_0^{2\pi} \sqrt{25(\sin^2 t + \cos^2 t)}$$

$$\int_0^{2\pi} 5\,dt = 5t\Big|_0^{2\pi} = \boxed{10\pi}$$

b) $\vec{r} = \vec{r_0} + \vec{v}\,t$

$\vec{r}'(0) = \langle 0, 0, 5 \rangle$

$\vec{r}(0) = \langle 3, 4, 0 \rangle$

$$\boxed{\vec{r} = \langle 3,4,0 \rangle + \langle 0,0,5 \rangle t}$$

6. (14 pts)

$$\frac{d(f \circ \vec{r})}{dt} = \nabla f(\vec{r}) \cdot \frac{d\vec{r}}{dt}$$

$\nabla f = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$ $\quad \vec{r}(1) = \langle 1, 2 \rangle$ $\quad \vec{r}' = \langle 3t^2, -\frac{2}{t^2} \rangle$

$\vec{r}'(1) = \langle 3, -2 \rangle$

6. (14 points)

$$\frac{dV}{dt} = V_r \frac{dr}{dt} + V_h \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi \cdot 2 \cdot 2(2) + \pi \cdot 2^2(3)$$

$$= 16\pi + 12\pi$$

$$= 28\pi$$