Solutions

MA 242 Test 4 Version 1

1. (14 points) Find the mass of the wire with density $\sigma(x,y,z) = x + yz$ in the shape of the line segment from (1,0,2) to (3,4,2)

2. (26 points) Use $\vec{\mathbf{F}}$(x,y,z)=$\langle y\cos(xy) + 3, x\cos(xy) + z, y + \frac{1}{2\sqrt{z}} \rangle$ to answer the following:
   a) Assume $\vec{\mathbf{F}}$(x,y,z) is path independent and find its most general potential function f
   b) Use the Fundamental theorem for Line Integrals to find the work done by $\vec{\mathbf{F}}$(x,y,z) moving a particle along the curve given by $\vec{\mathbf{r}}$(t)=$\langle t(t-3), t^2, t+1 \rangle$ where $0 \le t \le 3$

3. (20 points) Use $\vec{\mathbf{F}}$(x,y,z)=$\langle 2x\sqrt{z}, z, -y \rangle$ to answer the following:
   a) Show $\vec{\mathbf{F}}$(x,y,z)=$\langle 2x\sqrt{z}, z, -y \rangle$ is not conservative
   b) Find the work done by $\vec{\mathbf{F}}$(x,y,z) moving a particle along the curve C given by $\vec{\mathbf{r}}$(t)=$\langle t^2, t^3, t^2 \rangle$ where $0 \le t \le 3$

4. (14 points) Use Green's theorem to find the circulation of the vector field $\vec{\mathbf{F}}$(x,y)=$(\sqrt{x} + y^3)\mathbf{i}$ +(11tan(y)-x^3)$\mathbf{j}$ if C consists of the top half of the circle $x^2 + y^2 = 9$ from (-3,0) to (3,0) and the line segment from (3,0) to (-3,0)
   Hint: $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$

5. (16 points) Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ where $0 \le z \le 5$. Show your work!

6. (10 points) Find a parametric representation for the curve given by x=yln(y) in the plane z=7 from (0,1,7) to (3ln(3), 3,7) include bounds for t.

C3 T4 v1 Solutions

1. (14 points)

$\vec{r} = \langle 1,0,2\rangle(1-t) + \langle 3,4,2\rangle t$

$= \langle 1-t, 0, 2-2t\rangle + \langle 3t, 4t, 2t\rangle$

$= \langle 1+2t, 4t, 2\rangle \Big\} 4$

$\int_C x+yz\,ds = \int_0^1 \underbrace{1+2t+8t}_{4} \underbrace{\sqrt{2^2+4^2+0^2}}_{4}\,dt$

$\sqrt{20}\int_0^1 1+10t\,dt = \sqrt{20}\left[t+5t^2\right]_0^1$

$\boxed{\sqrt{20}\;6} \Big\} 2$

2. (26 points)

(14) a) $f_x = y\cos xy + 3$ $\quad f_y = x\cos xy + z \quad f_z = y + \frac{1}{2}z^{-1/2}$

$f = \sin xy + 3x + g(y,z)$

$f_y = x\cos xy + 0 + g_y$

$g_y = z$

$g = yz + h(z)$

$f = \sin xy + 3x + yz + h(z)$

$f_z = 0 + 0 + y + h'(z) =$

$h(z) = \sqrt{z} + C$

$\boxed{f = \overset{4}{\sin xy} + \overset{3}{3x} + \overset{3}{yz} + \overset{3}{\sqrt{z}} + \overset{2}{C}}$

(12) b) $\int \nabla f \cdot d\vec{r} = \underset{3}{f(\vec{r}(3)) - f(\vec{r}(0))} = f\underbrace{(0,9,4)}_{3} - f\underbrace{(0,0,1)}_{3} = 36+2-1 = \underbrace{\boxed{37}}_{3}$

3. (20 points)

a) $\text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x\sqrt{z} & z & -y \end{vmatrix}$

$4\,\} = \langle -1-1, -(0-\frac{x}{\sqrt{z}}), 0-0 \rangle \neq \vec{0}$

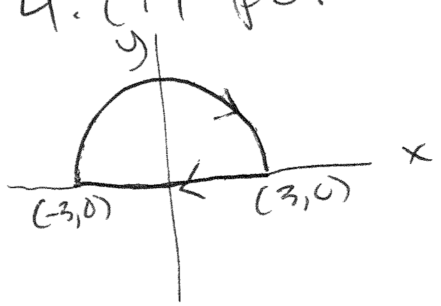
b) $W = \int_C \vec{F}\cdot d\vec{r} = \int_0^3 \underbrace{\langle 2t^2\sqrt{t^2}, t^2, -t^3\rangle}_{3} \cdot \vec{r}\,'$

$\int_0^3 \underbrace{\langle 2t^3, t^2, -t^3\rangle}_{2} \cdot \underbrace{\langle 2t, 3t^2, 2t\rangle}_{5}$

$\int_0^3 4t^4 + 3t^4 - 2t^4 \}3\,dt$

$\int_0^3 5t^4\,dt = \underbrace{t^5\Big|_0^3}_{2} = \underbrace{\boxed{3^5}}_{1}$

4. (14 points)



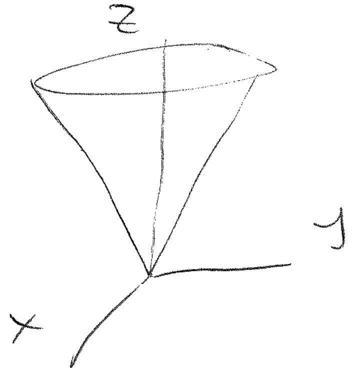
$\overset{2}{-}\iint \overbrace{-3x^2-3y^2}^{4}\,dA$

$-\int_0^{\pi}\int_0^{3} \overset{2}{\underbrace{-3r^2 r}}\,dr\,d\theta$

$\overset{2}{\pi}\,\frac{3}{4}r^4\Big|_0^3 \overset{3}{=} \underbrace{\boxed{\frac{3\pi}{4}\cdot 3^4}}_{1}$

5. (16 points)

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} \qquad \frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} \Big\} 4$$



$$3 \Big\{ \quad S.A. = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}$$

$$2 \Big\{ = \iint_D \sqrt{\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} + 1}\, dA$$

$$\iint_D \sqrt{1+1}\, dA$$

$$5 = \sqrt{x^2+y^2} = \sqrt{r^2} = r$$

$$\int_0^{2\pi}\int_0^5 \underbrace{\sqrt{2}}_{2}\, r\,dr\,d\theta$$

$$2\pi \sqrt{2}\, \frac{1}{2} r^2 \Big|_0^5$$

$$\boxed{\pi\sqrt{2}\, 25} \Big\} 5$$

6. (10 points)

$$\boxed{\begin{array}{ll} x = t\ln t & \} 3 \\ y = t & \} 1 \\ z = 7 & \} 3 \\ 1 \le t \le 3 & \} 3 \end{array}}$$