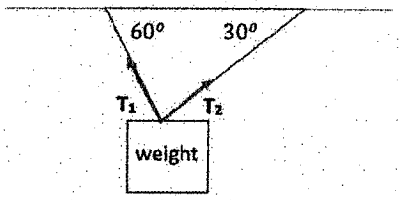
MA 242 Test 1 Version 2

1. (40 points) Use the points A(1,1,1), B(1, 4, 2), and C(2,1,3) to answer the following:
   a) Find a vector equation of the line segment AB
   b) Find vectors parallel to your line segment with a magnitude of 3
   c) Find the area of the triangle ABC
   d) Find the length of the side AB

2. (30 points) Use the intersecting lines L_1: x=1+3t, y=2t, z=t-2 and L_2: x=7+s, y=4, z=3s to answer the following:
   a) Find their point of intersection
   b) Find an equation of the plane containing these lines
   c) Find the angle between the lines using their direction vectors

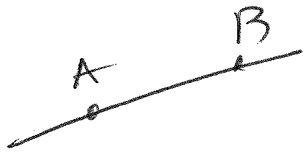
3. (15 points) A ball is thrown at an angle of elevation of 30° above the horizontal with an initial speed of 40m/s. Use is $\vec{a} = \langle 0,-10 \rangle$ for the acceleration due to gravity.
   a) Find the velocity vector $\vec{v}$
   b) Find the position vector $\vec{r}$
   c) Find its position at is maximum height

4. (15 points) Use the picture below to answer the following:
   a) The magnitude of tension vector $\mathbf{T_1}$ is 10 lb. Write tension vector $\mathbf{T_1}$ in its component form
   b) Find the magnitude of tension vector $\mathbf{T_2}$
   c) Find the weight of the object suspended by these cables



C3 T1 V2 Solutions

1. (40 points)



a) $\overrightarrow{AB} = \langle 1-1, 4-1, 2-1 \rangle = \langle 0, 3, 1 \rangle$

$$\boxed{\vec{r} = \langle 1,1,1 \rangle + \langle 0,3,1 \rangle t \;,\; 0 \le t \le 1}$$

b) $$\pm 3 \frac{\langle 0,3,1 \rangle}{\sqrt{10}}$$

c) $\overrightarrow{AC} = \langle 2-1, 1-1, 3-1 \rangle = \langle 1, 0, 2 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 1 \\ 1 & 0 & 2 \end{vmatrix}$$

$$= \langle 6, -(0-1), 0-3 \rangle$$

$$= \langle 6, 1, -3 \rangle$$

$$\frac{\|\overrightarrow{AB} \times \overrightarrow{AC}\|}{2} = \boxed{\frac{\sqrt{36+1+9}}{2}}$$

d) $\sqrt{10}$

2. (30 points)

a) $1+3t = 7+s$ ↖ $1+6 = 7+s$ $s=0$

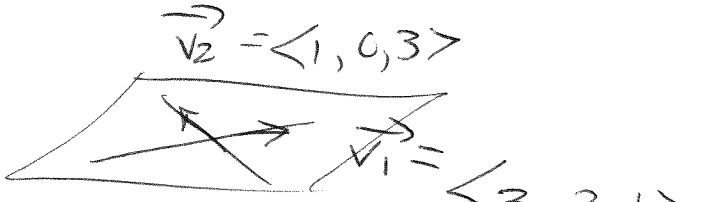
$2t = 4 \longrightarrow t=2$

$t-2 = 3s$

✓

pt: $(7,4,0)$

b) $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix}$



$= \langle 6, -(9-1), 0-2 \rangle$

$= \langle 6, -8, -2 \rangle$

$6(x-7) - 8(y-4) - 2(z-0) = 0$

c) $\langle 3,2,1 \rangle \cdot \langle 1,0,3 \rangle = \sqrt{9+4+1}\ \sqrt{1+0+9} \cos\theta$

$3+0+3$

$\theta = \cos^{-1}\left(\frac{6}{\sqrt{14}\ \sqrt{10}}\right)$

3. (15 points)

a) $\vec{v} = \langle 0, -10t \rangle + \vec{c}$

$\vec{v}(0) = \langle 40\cos 30^\circ, 40\sin 30^\circ \rangle$

$= \langle 40\frac{\sqrt{3}}{2}, 40(\frac{1}{2}) \rangle = \langle 20\sqrt{3}, 20 \rangle$

$\boxed{\vec{v} = \langle 20\sqrt{3}, 20 - 10t \rangle}$

b) $\boxed{\vec{r} = \langle 20\sqrt{3}\,t, 20t - 5t^2 \rangle} + \cancel{\vec{d}}$

c) $20 - 10t = 0 \quad t = 2$

$\vec{r}(2) = \boxed{\langle 40\sqrt{3}, 40 - 20 \rangle}$

4. (15 points)

$\vec{T_1} = \langle -10\cos 60^\circ, 10\sin 60^\circ \rangle$

$= \langle -10\,\frac{1}{2}, 10\,\frac{\sqrt{3}}{2} \rangle$

a)

$\vec{T_1} = \langle -5, 5\sqrt{3} \rangle$

b) $\vec{T_1} + \vec{T_2} = \langle 0, \text{weight} \rangle$

$\langle -5, 5\sqrt{3} \rangle + \langle T_2\cos 30^\circ, T_2 \sin 30^\circ \rangle = \langle 0, \text{weight} \rangle$

$-5 + T_2\cos 30^\circ = 0$

$-5 + T_2\,\frac{\sqrt{3}}{2} = 0$

$T_2 = \frac{10}{\sqrt{3}}$

c) $5\sqrt{3} + T_2 \sin 30^\circ = \text{weight}$

$5\sqrt{3} + \frac{10}{\sqrt{3}}\,\frac{1}{2} = \text{weight}$

$5\sqrt{3} + \frac{5}{\sqrt{3}}$