MA 242 Test 3 Version 1

1. (12 points) Evaluate the iterated integral $\int_0^2 \int_0^{4-x^2} \frac{2x\cos(3y)}{4-y}\,dy\,dx$ by switching the order of integration.

2. (14 points) Find the average value of $f(x,y)=e^y$ over the triangular region in the xy-plane with vertices (0,0), (2,6), and (2,0)

3. (17 points) Evaluate $\iiint_F y^2 dV$ if F is the solid bounded by the plane z=18 and the paraboloid $z = 2x^2 + 2y^2$
   Hint: You may wish to reference one or more of the following identities
   $\cos^2 u = \frac{1}{2}(1 + \cos(2u))$
   $\sin^2 u = \frac{1}{2}(1 - \cos(2u))$

4. (12 points) **Set up** the integral needed to find the mass of the solid tetrahedron in the 1st octant bounded above by the plane z=12-6x-2y and below by z=2 if its density is $\sigma(x,y,z) = 1 + xz$ **Do not evaluate.**

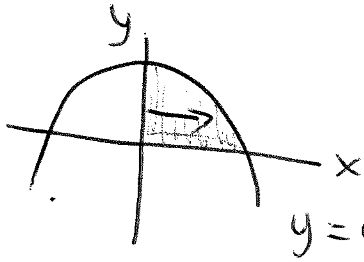
5. (16 points) Use spherical coordinates to **set up** the iterated integral $\iiint_F y\,(x^2 + y^2 + z^2)dV$ if F is the solid that lies inside the sphere $x^2 + y^2 + z^2 = 25$ in the 1st octant. **Do not evaluate.**

6. (15 points) Find the mass of the lamina bounded by the region D where D= $\{(x,y) | 1 \le x^2 + y^2 \le 9,\ 0 \le y \le x\}$ with density $\sigma(x,y) = \frac{1}{x^2+y^2}$

7. (14 points) **Set up** the double integral needed to find the volume of solid F that lies inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 4$ **Do not evaluate.**

C3V1 Solutions

1. (12 points)



$$\int_0^4\int_0^{\sqrt{4-y}} \frac{2x\cos(3y)}{4-y}\,dx\,dy$$

$y=4-x^2$

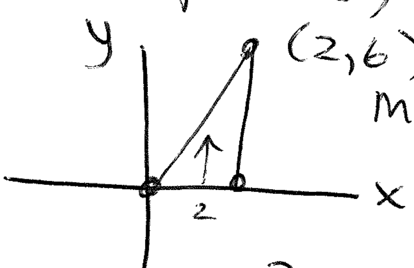
$x^2=4-y$

$$=\int_0^4 \frac{x^2\cos(3y)}{4-y}\Big|_0^{\sqrt{4-y}}\,dy$$

$$=\int_0^2 \frac{\cancel{(4-y)}\cos(3y)}{\cancel{4-y}}\,dy = \frac{1}{3}\sin 3y\Big|_0^2$$

$$\boxed{\frac{1}{3}\sin 6}$$

2. (14 points)

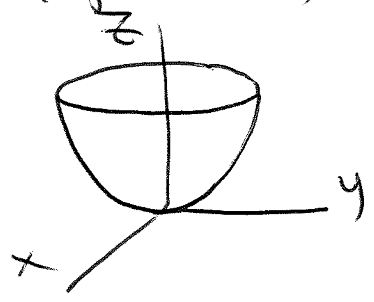


$$m=\frac{\Delta y}{\Delta x}=\frac{6-0}{2-0}=3$$

$$\frac{\int_0^2\int_0^{3x} e^y\,dy\,dx}{\frac{1}{2}\cdot 2\cdot 6} = \frac{\int_0^2 e^{3x}-\overset{1}{\cancel{e^0}}\,dx}{6}$$

$$=\frac{\frac{1}{3}e^{3x}-x\Big|_0^2}{6} = \boxed{\frac{\frac{1}{3}e^{6}-2-\frac{1}{3}}{6}}$$

3. (17 points)



$$\int_0^{2\pi}\int_0^{3}\int_{2r^2}^{18} r^2\sin^2\theta \, dz\, r\, dr\, d\theta$$

$18 = 2r^2$

$9 = r^2$

$$\int_0^{2\pi} \tfrac{1}{2}(1-\cos 2\theta)\, d\theta \int_0^3 r^2 z \Big|_{2r^2}^{18} r\, dr$$

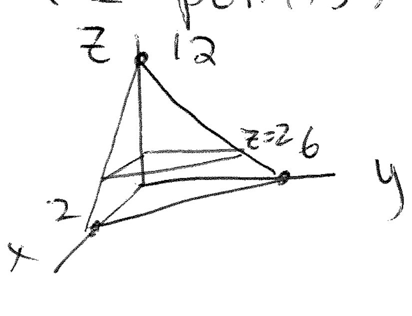
$$\tfrac{1}{2}\left(\theta - \tfrac{1}{2}\sin 2\theta\right)_0^{2\pi} \int_0^3 r^3(18-2r^2)\, dr$$

$$\tfrac{1}{2}(2\pi) \int_0^3 18r^3 - 2r^5\, dr$$

$$\pi\left[\frac{18}{4}r^4 - \frac{2}{6}r^6\right]_0^3$$

$$\boxed{\pi\left[\frac{18}{4}\cdot 3^4 - \frac{2}{6}\cdot 3^6\right]}$$

4. (12 points)



$$m = \int_0^{5/3}\int_0^{5-3x}\int_2^{12-6x-2y} 1 + xz \, dz\, dy\, dx$$

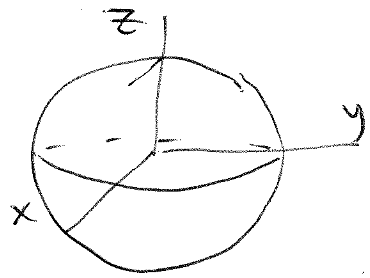
$12 - 6x - 2y = 2$ $\qquad 10 - 6x = 2y$

$\cancel{2y = 10-6x}$ $\qquad y = 5 - 3x$

$5 - 3x = 0$

$x = 5/3$

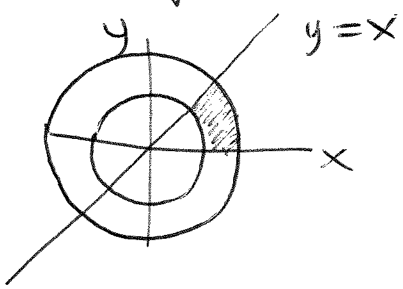
5. (16 points)



1st octant

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{5} \rho\sin\phi\sin\theta\ \rho^2\ \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

6. (15 points)



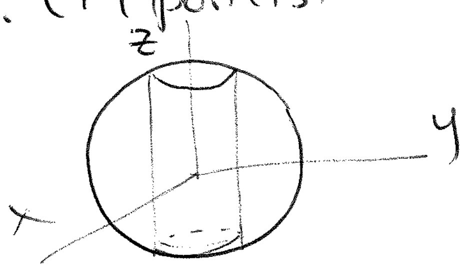
$$m = \iint \frac{1}{x^2+y^2}\, dA$$

$$= \int_0^{\pi/4}\int_1^{3} \frac{1}{r^2}\, r\, dr\, d\theta$$

$$\frac{\pi}{4}\int_1^{3} \frac{1}{r}\, dr$$

$$\frac{\pi}{4} \ln r \Big|_1^3 = \frac{\pi}{4}\left(\ln 3 - \cancel{\ln 1}\right)$$

7. (14 points)



$$2\int_0^{2\pi}\int_2^{3} \sqrt{9-r^2}\ r\, dr\, d\theta$$