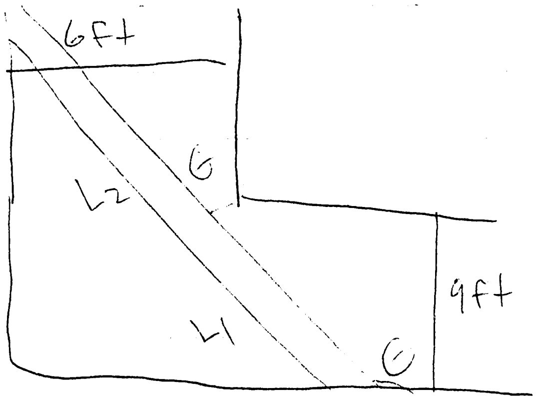
Ex 40. A steel pipe is being carried down a hallway 9ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 6ft wide.
What is the length of the longest pipe that can be carried horizontally?



$\sin\theta = \frac{6}{L_2}$ $\quad L_2 = \frac{6}{\sin\theta} = 6\csc\theta$

$\cos\theta = \frac{9}{L_1}$ $\quad L_1 = 9\sec\theta$

$L = L_1 + L_2 = 9\sec\theta + 6\csc\theta$

$L' = 9\sec\theta\tan\theta - 6\csc\theta\cot$

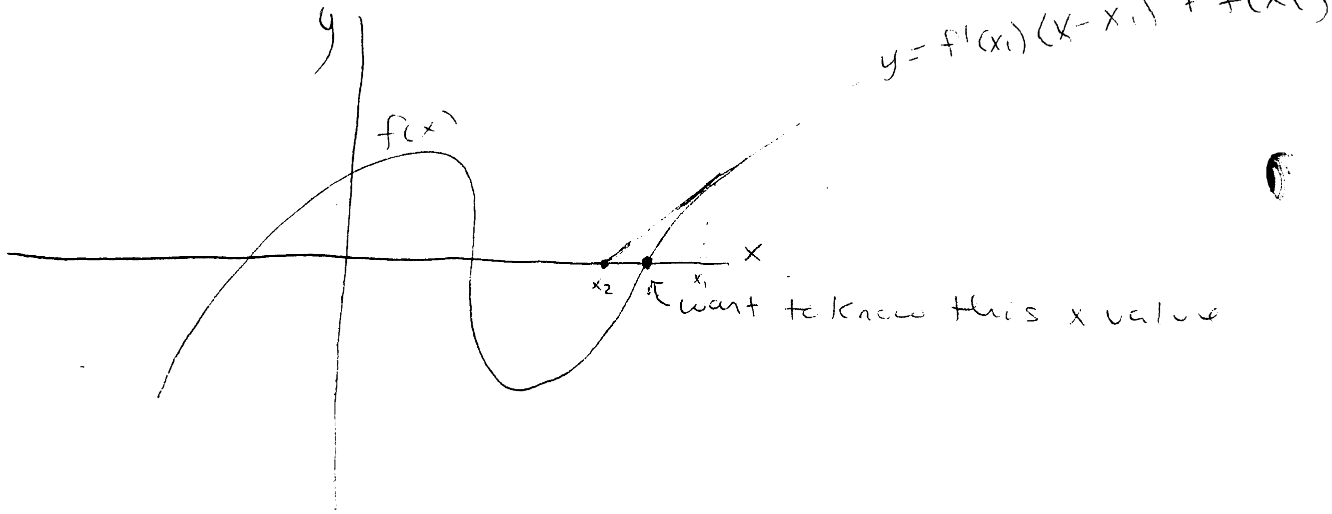
$L' = 0 \quad \tan\theta = \sqrt[3]{2/3} \quad L = 21\ \text{ft.}$

4.8 Newton's Method

Newton's Method is a technique for approximating solutions that require a high level of computational difficulty. This is usually used to find roots of a problem.

Technique: 1. Set x_1 = a number close to the solution.

2. $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$



Proof: $0 = f'(x_1)(x - x_1) + f(x_1)$

$$f'(x_1)x - f'(x_1)x_1 = -f(x_1)$$

$$f'(x_1)x = f'(x_1)x_1 - f(x_1)$$

$$x = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Call $x = x_2$

Notice that x_2 is closer to our solution than x_1 is.

In general,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where $\lim_{n\to\infty} x_n = r$

Example: Use Newton's Method to approximate $\sqrt{2} \approx 1.414213562$

$x = \sqrt{2}$

$x^2 = 2$

$x^2 - 2 = 0$

$f(x) = x^2 - 2$

$f'(x) = 2x$

$x_1 = 3/2$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

To save time, if you have a graphing Calculator:

* Enter the value of x_1 into your calculator.
* Then on a new line input:

$$Ans - \frac{f(Ans)}{f'(Ans)}$$

(for this 1st example it would be

$$Ans - \frac{(Ans^2 - 2)}{(2*Ans)}$$
)

* Press enter, this is your x_2
  (In this example $x_2 = 1.416\overline{6}$)
* Then hit the "entry" key (on my calculator you would hit $\boxed{2nd}$ enter)
* Then hit enter to find x_3
  (In this case 1.414215686)
* Repeat the steps above to get to the specified x_n or until your x's stop changing

$x_4 = 1.414213562$

$x_5 = 1.414213562$

Ex p326 Use Newtons Method with
#5 $x_1 = 1$ to find x_3

$x^3 + 2x - 4 = 0$

$f(x) = x^3 + 2x - 4 \qquad f'(x) = 3x^2 + 2$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.2 \qquad x_3 = 1.179746835$

$f(x_3) = .001468$

Ex Use Newton's Method to find an approximation of the root of the equation $x^5 - 7x^4 + 18x^2 - 17x + 13 = 0$ on the interval $[-2, -1.8]$ up to 6 decimal places.

$f(x) = x^5 - 7x^4 + 18x^2 - 17x + 13$

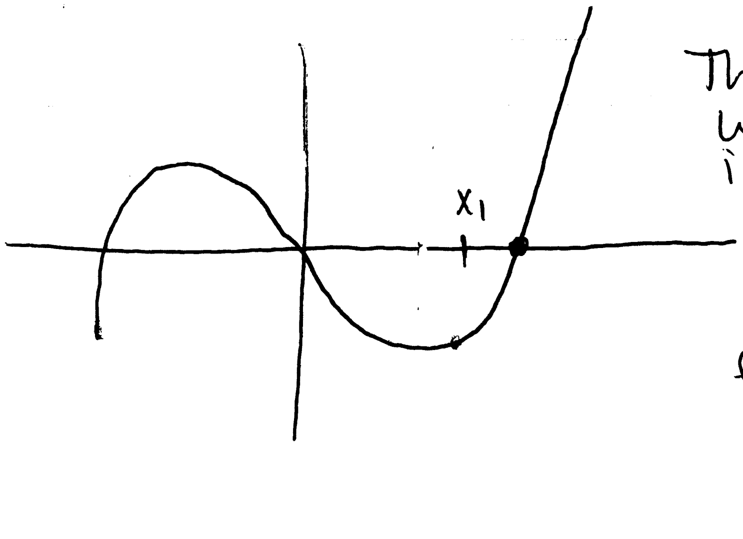
$f'(x) = 5x^4 - 28x^3 + 36x - 17$

$x_1 = -1.8$

$x_2 = -1.8 - \dfrac{f(-1.8)}{f'(-1.8)} =$

Be Careful: If $f'(x_1)$ is close to zero your x_n's might not approach any number. If this is the case choose another x_1.

Ex



Think about what would happen if this was our x_1. Notice we aren't getting closer to finding our root.