Calculus I Test 1 Version 1 No work = No credit!

Calculators are not permitted on this or any test.

1. (15 points) Use $f(x) = \dfrac{x+3}{2x+1}$ to answer the following

   a) Find the domain of $f(x)$

   b) Find $f^{-1}(x)$

   c) Find the domain and range of $f^{-1}(x)$

2. (10 points) Solve the following for x

   a) $3^{x-2} - 5 = 0$

   b) $\log_x 8 = 3$

3. (15 points) Consider the parametric curve:

   $$x(t) = \sin(t) - 4 \quad y(t) = 3 + \cos(t)$$

   a) Eliminate the parameter t and find a Cartesian equation

   b) Identify the curve

   c) For $0 \le t \le \pi$, sketch the graph of the curve. Clearly label and give coordinates for the initial point, the terminal point, and one other point on the curve. Draw an arrow to show increasing t.

4. (10 points) Use the Squeeze theorem to find $\lim_{x \to 0} x^3 \sin\left(\dfrac{1}{x}\right)$

5. (25 points) Evaluate the following limits using the techniques we've developed in class (Derivatives are NOT permitted!)

   a) $\lim_{x \to 2} x^2 - 5x + e^x$

   b) $\lim_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

   c) $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

TURN OVER

6. (10 points) Find the equation of the line through the points (1,-2) and (3,4). Simplify it to slope - intercept form.

7. (15 points) Use the graph of f(x) given below to find the following.

Only answers in blue books will be graded.

a) $\lim_{x \to (-3)^-} f(x)$

b) f(-3)

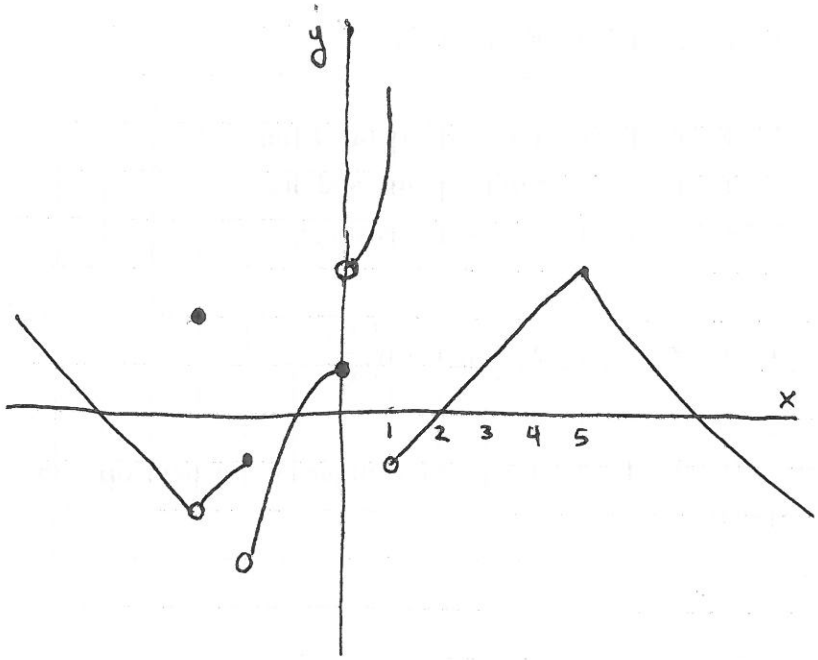
c) $\lim_{x \to (-3)} f(x)$

d) $\lim_{x \to 0^+} f(x)$

e) $\lim_{x \to 1} f(x)$

f) f(5)

g) List the values of x where f is discontinuous. State the type of discontinuity at each of those values.



C1 T1 V1 Solutions

1. (15 pts)

a) $2x+1=0$

$x \neq -1/2$

b) $y = \dfrac{x+3}{2x+1}$

$y(2x+1) = x+3$

$2xy + y = x+3$

$2xy - x = 3-y$

$x(2y-1) = 3-y$

$x = \dfrac{3-y}{2y-1}$

$y = \dfrac{3-x}{2x-1}$

$$\boxed{f^{-1}(x) = \frac{3-x}{2x-1}}$$

c) Domain $x \neq 1/2$

Range $y \neq -1/2$

2. (10 points)

a) $3^{x-2} - 5 = 0$

$3^{x-2} = 5$

$x - 2 = \log_3 5$

$x = 2 + \log_3 5$

b) $\log_x 8 = 3$

$x^3 = 8$

$\boxed{x = 2}$

3. (15 pts)

a) $x + 4 = \sin t$ $\qquad$ $y - 3 = \cos t$

$(x+4)^2 = \sin^2 t$ $\qquad$ $(y-3)^2 = \cos^2 t$

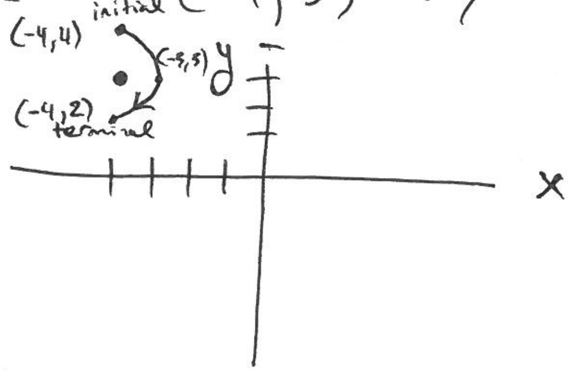
$\sin^2 t + \cos^2 t = 1$

$\boxed{(x+4)^2 + (y-3)^2 = 1}$

b) Circle centered at $(-4, 3)$ w/ radius $= 1$

c)

t	x	y
0	-4	4
$\frac{\pi}{2}$	-3	3
π	-4	2



4. (10 pts) $-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$$-x^3 \le x^3 \sin\left(\frac{1}{x}\right) \le x^3$$

$$\lim_{x \to 0} -x^3 = 0$$

$$\lim_{x \to 0} x^3 = 0$$

$\lim_{x \to 0} x^3 \sin\left(\frac{1}{x}\right) = 0$ by Squeeze thm

5. (25 pts)

a) $\lim_{x \to 2} x^2 - 5x + e^x = 4 - 10 + e^2 = -6 + e^2$

b) $\lim_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h} = \lim_{h \to 0} \dfrac{\frac{1}{3+h} - \frac{1}{3}}{h}$

$= \lim_{h \to 0} \dfrac{\frac{3}{3(3+h)} - \frac{(3+h)}{3(3+h)}}{h} = \lim_{h \to 0} \dfrac{\frac{\cancel{3}-\cancel{3}-h}{3(3+h)}}{h}$

$= \lim_{h \to 0} \dfrac{-\cancel{h}}{3(3+h)\cancel{h}}$

$= \boxed{\dfrac{-1}{9}}$

c) $\lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}}$

$= \lim_{x \to 16} \frac{16-x}{(16x-x^2)(4+\sqrt{x})}$

$= \lim_{x \to 16} \frac{\cancel{16-x}}{x\cancel{(16-x)}(4+\sqrt{x})}$

$= \frac{1}{16(4+\sqrt{16})} = \boxed{\frac{1}{16(8)}}$.

6. (10 pts)

$m = \frac{\Delta y}{\Delta x} = \frac{4-(-2)}{3-1} = \frac{6}{2} = 3$

$y - y_1 = m(x - x_1)$

$y - 4 = 3(x-3)$

$y = 3x - 9 + 4$

$\boxed{y = 3x - 5}$

7. (15 pts)

a) -2

b) 2

c) -2

d) 3

e) DNE

f) 3

g) $x=-3$ removeable discontinuity

$x=-2$ jump discontinuity

$x=0$ jump discontinuity

$x=1$ infinite discontinuity