MA 141 - 002 Test 4 - Version 2 SOLUTIONS

SHOW ALL OF YOUR WORK!!!!

1. (10 points) Find the most general antiderivative of the function

$$f(x) = 5x^{\frac{3}{7}} - \sec(x)\tan(x) + \frac{1}{x} - 12x^{-7} + \pi^3$$

$$F(x) = \frac{5x^{\frac{3}{7}+\frac{7}{7}}}{\frac{10}{7}} - \sec x + \ln|x| + 2x^{-6} + \pi^3 x + C$$

$$= \boxed{\frac{7}{2}x^{\frac{10}{7}} - \sec x + \ln|x| + 2x^{-6} + \pi^3 x + C}$$

2. (10 points) Evalutate the Riemann Sum of $f(x) = x^3 + 4$ where $0 \le x \le 6$ with three subintervals, taking the sample points to be left endpoints.

$$\Delta x = \frac{6-0}{3} = 2$$

0 2 4 6

$$L_3 = 2(f(0) + f(2) + f(4))$$

$$= \boxed{2(4 + 2^3 + 4 + 4^3 + 4)}$$

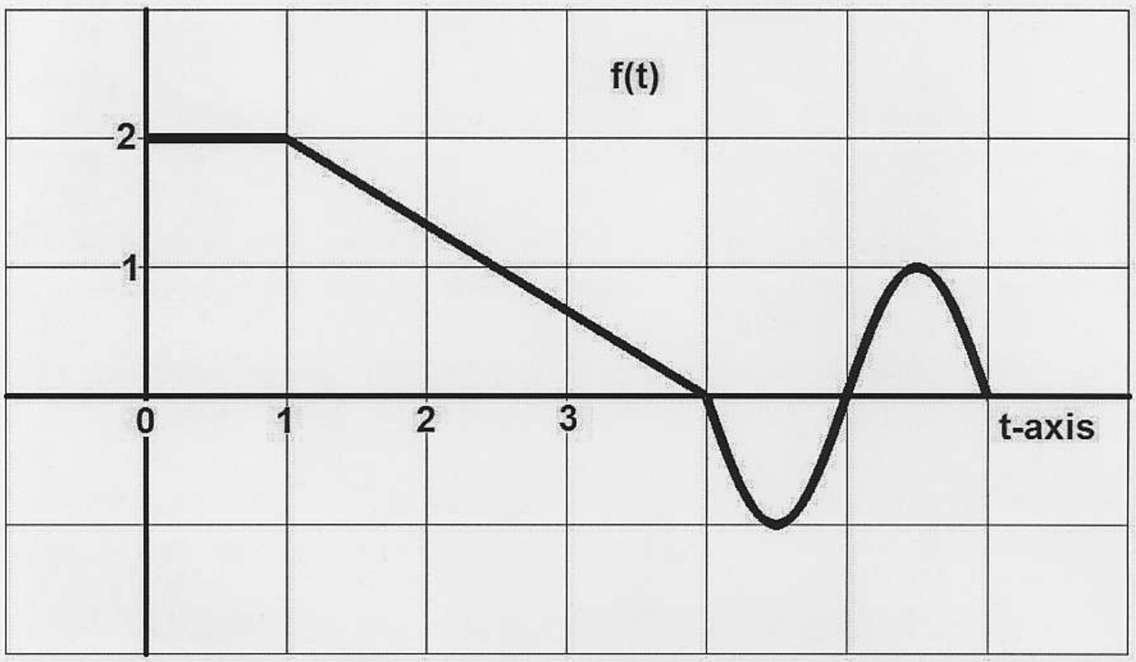
3. (10 points) a) If $g(x) = \int_{-4}^{x} \frac{\sec(t^9)(t - 13t^2)}{\sin(t^2)}\,dt$, find $g'(x)$.

$$g'(x) = \frac{\sec x^9 (x - 13x^2)}{\sin(x^2)}$$

b) What theorem did you use to find this?

Fundamental Theorem of Calculus

4. (20 points) If $g(x) = \int_0^x f(t)dt$, use the graph of f(t) below to answer the following questions.



a) Find g(0) $= 0$

b) Find g(1) $= 2$

c) Find g(4) $= 2 + 3 = 5$

d) Find g′(x) $= f(x)$

e) Find g′(0) $= f(0) = 2$

f) Where is g increasing? $(0,4)$ $(5,6)$

g) Where is g decreasing? $(4,5)$

h) Where does g have a local maximum? $x = 4$

i) Where does g have a local minimum? $x = 5$

j) Find $\int_4^6 f(t)dt$ $= 0$

5. (13 points) Evaluate the integral $\int_0^{\frac{\pi}{2}} \frac{\text{sinx}}{1+\cos^2 x}dx$

$u = \cos x$

$du = -\sin x \, dx$

$u(0) = \cos 0 = 1$

$u(\frac{\pi}{2}) = 0$

$$= \int_1^0 \frac{-du}{1+u^2} = -\tan^{-1} u \Big|_1^0$$

$$= -\tan^{-1} 0 + \tan^{-1} 1$$

$$= \boxed{\frac{\pi}{4}}$$

6. (13 points) Evaluate the integral $\int_1^3 \text{xlnxdx}$

LIATE

$u = \ln x \qquad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx \quad dv = x\,dx$

$$\frac{1}{2}x^2 \ln x \Big|_1^3 - \int_1^3 \frac{1}{2}x^2 \frac{1}{x} dx$$

$$\frac{1}{2}x^2 \ln x \Big|_1^3 - \int_1^3 \frac{1}{2}x\,dx = \frac{9}{2}\ln 3 - \cancel{\frac{1}{2}\ln 1} - \frac{1}{4}x^2 \Big|_1^3$$

$$= \frac{9}{2}\ln 3 - \frac{9}{4} + \frac{1}{4} = \frac{9}{2}\ln 2 - \frac{8}{4} = \boxed{\frac{9}{2}\ln 2 - 2}$$

7. (12 points) Evaluate the integral $\int \cos^3 x \sin^5 x dx$

$$\int \cos^3 x \sin^5 x \, dx = \int \cos x \cos^2 x \sin^5 x \, dx$$

$$= \int \cos x (1 - \sin^2 x) \sin^5 x \, dx \qquad u = \sin x \quad du = \cos x \, dx$$

$$= \int (1-u^2) u^5 du = \int u^5 - u^7 du = \frac{1}{6}u^6 - \frac{1}{8}u^8 + C$$

$$= \boxed{\frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C}$$

8. (12 points) Evaluate the integral $\int x^2 e^{3x}\,dx$

LIATE

sign	u & its derivatives	dV & its antiderivatives
+	x^2	e^{3x}
−	$2x$	$\frac{1}{3}e^{3x}$
+	2	$\frac{1}{9}e^{3x}$
−	0	$\frac{1}{27}e^{3x}$

$$= \boxed{\frac{x^2}{3}e^{3x} - \frac{2x}{9}e^{3x} + \frac{2}{27}e^{3x} + C}$$

Extra Credit : (5 points) Using techniques developed in this class, solve the following problem.
A stone was dropped off a cliff and hit the ground with a velocity of -120 ft/s.
What is the height of the cliff? Hint : Acceleration due to gravity = -32ft/s^2

$a = -32$

$v = -32t + C$ (Stone dropped $\rightarrow C = 0$)

$V = -32t$

$p = -16t^2 + D$

When the stone hits the ground position = 0 and the velocity = -120 ft/s so we can solve for t $-120 = -32t$ when the stone hits the ground $\rightarrow t = \frac{15}{4}$

$P(\frac{15}{4}) = -16(\frac{15}{4})^2 + D = 0 = -225 + D = 0 \rightarrow$

$D = 225$

Height of the cliff $\boxed{225 \text{ft}}$