MA 242 Test 4 Practice Problems: This should be a part of your Test 4 preparation. It doesn't replace timing yourself taking the Practice Test, working additional problems from the review sheet, or looking over the worksheets. Your test will possibly have matching with vector fields (including gradient vector fields).

These problems are mostly inspired by Calculus by Edwards & Penney.

1. Find the mass of the wire with density $\sigma(x,y) = x^2 + y^2$ in the shape of the curve C given by $\mathbf{r}(t) = (4t - 1)\mathbf{i} + (3t + 1)\mathbf{j}$ where $-1 \le t \le 1$

2. Find the line integral with respect to arc length of $f(x,y,z) = 2x - y$ where C is the circle $x^2 + y^2 = 25$ that lies in the plane $z = 4$

3. Find the work done by the vector field $\vec{\mathbf{F}}(x,y) = \langle x^2y, xy^3 \rangle$ moving a particle along the line segment (-1,1) to (2,1) and then along the line segment from (2,1) to (2,5)

4. Evaluate $\int_c \vec{\mathbf{F}} \bullet d\mathbf{r}$ where $\vec{\mathbf{F}}(x,y,z) = \langle 2x + 3y, 3x + 2y, 3z^2 \rangle$ moving a particle along C where C is the path from (0,0,0) to (4,2,3) that consists of three line segments parallel to the x - axis, the y - axis, and the z - axis in that order.
Hint : There is an efficient way to do this problem

5. Use $\vec{\mathbf{F}}(x,y) = (x + \arctan(y))\mathbf{i} + \left(\frac{x + y}{1 + y^2}\right)\mathbf{j}$ to answer the following.
Determine if $\vec{\mathbf{F}}$ is conservative. If it is find its most general potential function f

6. Use $\vec{\mathbf{F}}(x,y) = (e^x \sin y + \tan y)\mathbf{i} + \left(e^x \cos y + x\sec^2 y + \frac{3}{1 + y}\right)\mathbf{j}$ to answer the following.
Determine if $\vec{\mathbf{F}}$ is conservative. If it is find its most general potential function f

7. Show that the given line integral is independent of path, then calculate the value of the line integral $\int_c \vec{\mathbf{F}} \bullet d\mathbf{r}$ where $\vec{\mathbf{F}}(x,y,z) = \left(-\sin(x + 2z) + \frac{1}{\sqrt{x}}\right)\mathbf{i} + \left(3ze^{3yz}\right)\mathbf{j} + \left(-2\sin(x + 2z) + 3ye^{3yz} + z^2\right)\mathbf{k}$ along any smooth curve from (4,0,-2) to (1,1,1)

8. Find the area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$

9. Find the area that is cut from the surface $z = xy$ by the cylinder $x^2 + y^2 = 36$

10. Evaluate the surface integral $\iint_S f(x,y,z)\,dS$ where $f(x,y,z) = xyz$ and S in the triangle with vertices (3,0,0), (02,0) and (0,0,6)

11. Evaluate the surface integral $\iint_S f(x,y,z)dS$ where f(x,y,z) $= z^2$ and S in the part of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 4$

12. Find the mass of the surface S where S is the part of the sphere $x^2 + y^2 + z^2 = 25$ that lies above the plane z = 3 if it has density $\sigma(x,y,z) = x^2 + y^2$