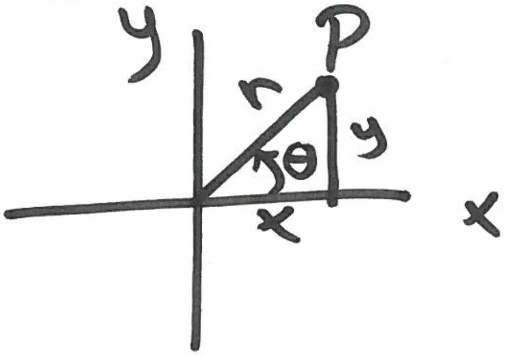
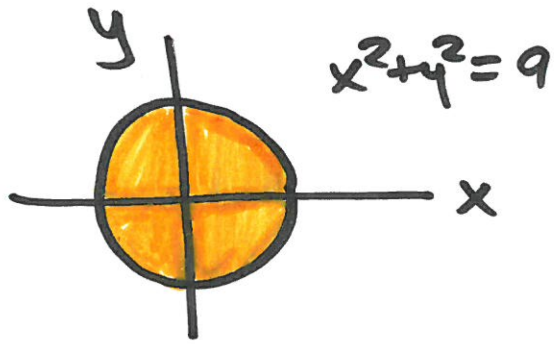
Polar Coordinate Bounds

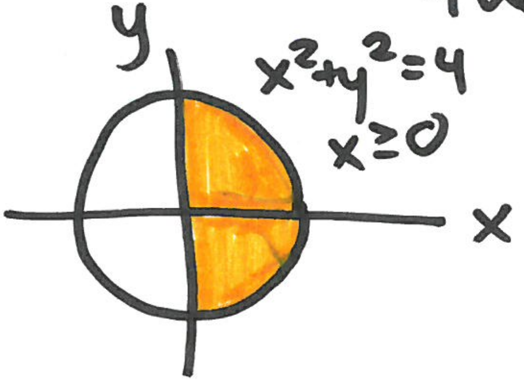


θ goes from the positive x-axis to our line segment from the origin to point P

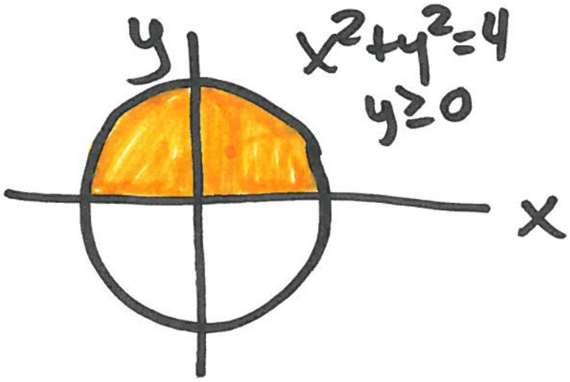


$$\int_0^{2\pi}\int_0^3 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

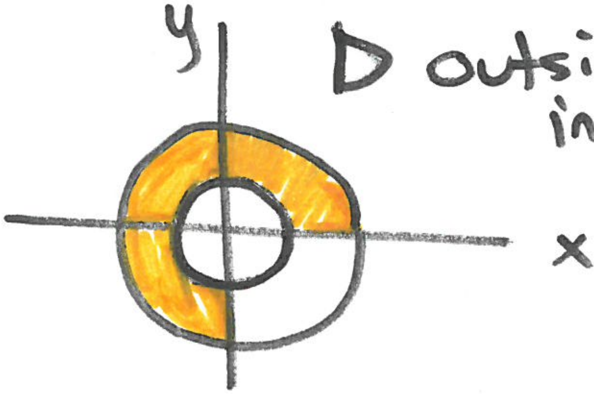
whole circle centered at the origin



$$\int_{-\pi/2}^{\pi/2}\int_0^2 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

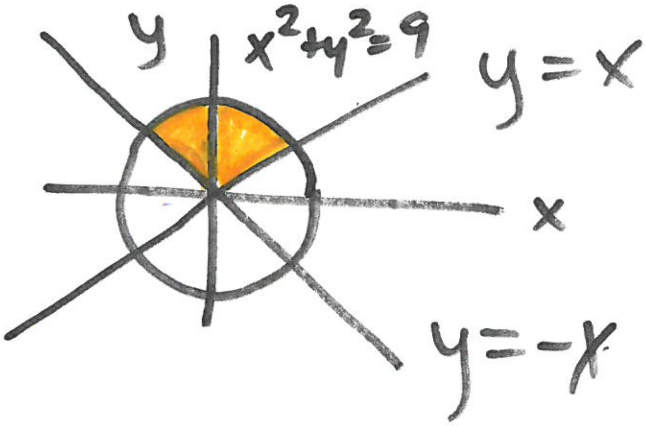


$$\int_0^{\pi}\int_0^2 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

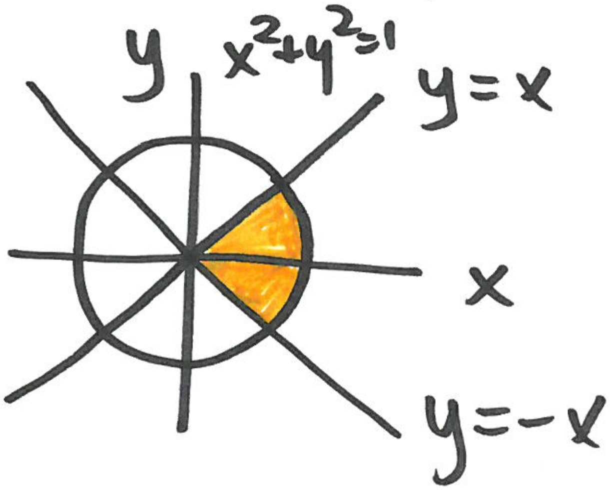


D outside $x^2+y^2=1$
inside $x^2+y^2=4$

$$\int_0^{\frac{3\pi}{2}}\int_1^2 f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$



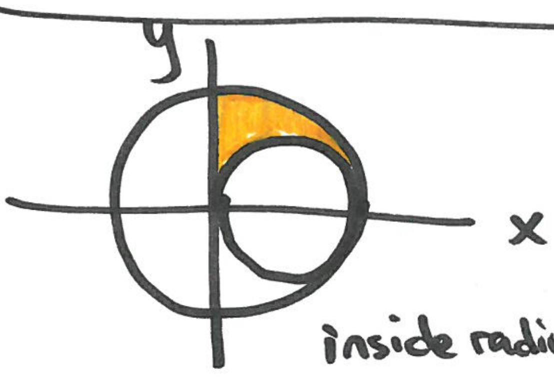
$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_0^3 f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$



$$\int_{-\frac{\pi}{4}}^{\pi/4}\int_0^1 f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

$$\boxed{\begin{array}{l}\text{Inside } x^2+y^2=4\\ \text{Outside } x^2+y^2=2x\\ \text{in the first octant}\end{array}}$$

For graphing:
$x^2+y^2-2x=0$
$(x-1)^2+y^2=1$



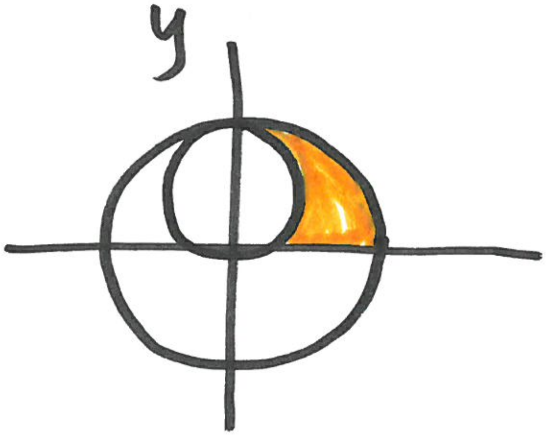
$$\int_0^{\pi/2}\int_{2\cos\theta}^{2} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

outside radius fixed

inside radius is changing: $x^2+y^2=2x \rightarrow r^2=2r\cos\theta$
$r=2\cos\theta$

Inside $x^2+y^2=4$
Outside $x^2+y^2=2y$
$x\ge 0,\ y\ge 0$

For Graphing:
$\rightarrow$ $x^2+y^2-2y=0$
$x^2+(y-1)^2=1$



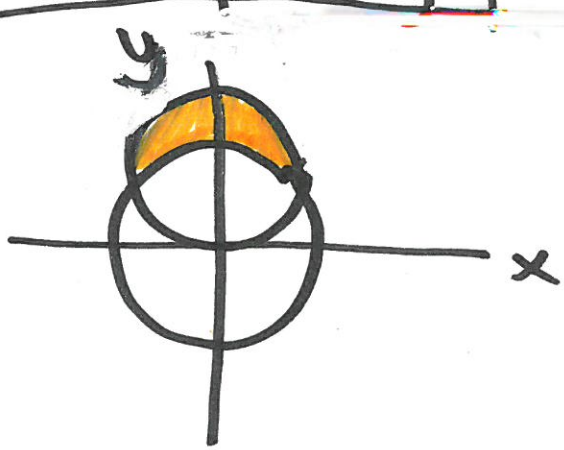
$$\int_0^{\pi/2}\int_{2\sin\theta}^{2} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

$x^2+y^2=2y$
$r^2=2r\sin\theta$

We can use the fact that for $x^2+y^2=4$ w/ $x\ge 0, y\ge 0$, $0\le\theta\le\frac{\pi}{2}$

Outside $x^2+y^2=1$
Inside $x^2+y^2=2y$

$$\int_{\pi/6}^{\pi-\frac{\pi}{6}}\int_{1}^{2\sin\theta} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$



For θ we want where the circles intersect

$r^2=1$ & $r^2=2r\sin\theta$

$1=2r\sin\theta$ $\leftarrow$ $r=1$ since $r^2=1$

$\frac{1}{2}=\sin\theta$ $\quad \theta=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$