P394

Find the inverse laplace

15. $\dfrac{se^{-3s}}{s^2+4s+5} = e^{-3s}\left(\dfrac{s}{s^2+4s+5}\right) = e^{-3s}\left(\dfrac{A(s-\alpha)+B\beta}{(s-\alpha)^2+\beta^2}\right)$

$$s^2+4s+5 = (s-\alpha)^2+\beta^2$$

$$s^2-2\alpha s+\alpha^2+\beta^2$$

$$\alpha=-2 \quad \beta=1$$

$$= e^{-3s}\left(\frac{A(s+2)+B}{(s+2)^2+1}\right)$$

$$\frac{A(s+2)+B}{(s+2)^2+1} = \frac{s}{(s+2)^2+1}$$

$$As+2A+B = s$$

$$A=1 \quad B=-2$$

$$= e^{-3s}\left(\frac{1(s+2)-2}{(s+2)^2+1}\right)$$

$$F(s) = \frac{1(s+2)}{(s+2)^2+1} - \frac{2}{(s+2)^2+1} \longrightarrow f(t) = 1e^{-2t}\cos t - 2e^{-2t}\sin t$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\left(\frac{(s+2)-2}{(s+2)^2+1}\right)\right\} = f(t-3)\,u(t-3)$$

$$= \left[e^{-2(t-3)}\cos(t-3) - 2e^{-2(t-3)}\sin(t-3)\right]u(t-3)$$

17. $\dfrac{e^{-3s}(s-5)}{(s+1)(s+2)} = e^{-3s}\left(\dfrac{A}{s+1} + \dfrac{B}{s+2}\right)$

$A(s+2) + B(s+1) = s-5$

$\underline{As} + 2A + \underline{Bs} + B = s-5$

$A + B = 1$

$-(2A + B = -5)$

$-A = 6 \quad A = -6 \quad B = 7$

$= e^{-3s}\left(\dfrac{-6}{s+1} + \dfrac{7}{s+2}\right)$

$\qquad\qquad ''$

$\qquad F(s) \longrightarrow f(t) = -6e^{-t} + 7e^{-2t}$

$\mathcal{L}^{-1}\left\{e^{-3s}\left(\dfrac{-6}{s+1} + \dfrac{7}{s+2}\right)\right\} = u(t-3)f(t-3)$

$= u(t-3)\left[-6e^{-(t-3)} + 7e^{-2(t-3)}\right]$

29. $y'' + y = u(t-3)$ $\quad y(0) = 0 \quad y'(0) = 1$

$$s^2\mathcal{L}\{y\} - \cancel{sy(0)} - y'(0) + \mathcal{L}\{y\} = \frac{e^{-3s}}{s}$$

$$\mathcal{L}\{y\}(s^2+1) = \frac{e^{-3s}}{s} + 1$$

$$\mathcal{L}\{y\} = e^{-3s}\left(\frac{1}{s(s^2+1)}\right) + \frac{1}{s^2+1}$$

$$\frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$\alpha = 0,\ \beta = 1$

$$As^2 + A + Bs^2 + Cs = 1$$

$A + B = 0$

$C = 0$

$A = 1 \rightarrow B = -1$

$$\mathcal{L}\{y\} = e^{-3s}\left(\frac{1}{s} \quad \frac{-s}{s^2+1}\right) + \frac{1}{s^2+1}$$

$f(t) = 1 - \cos t$

$$y = u(t-3)f(t-3) + \sin t$$

$$y = u(t-3)\left[1 - \cos(t-3)\right] + \sin t$$