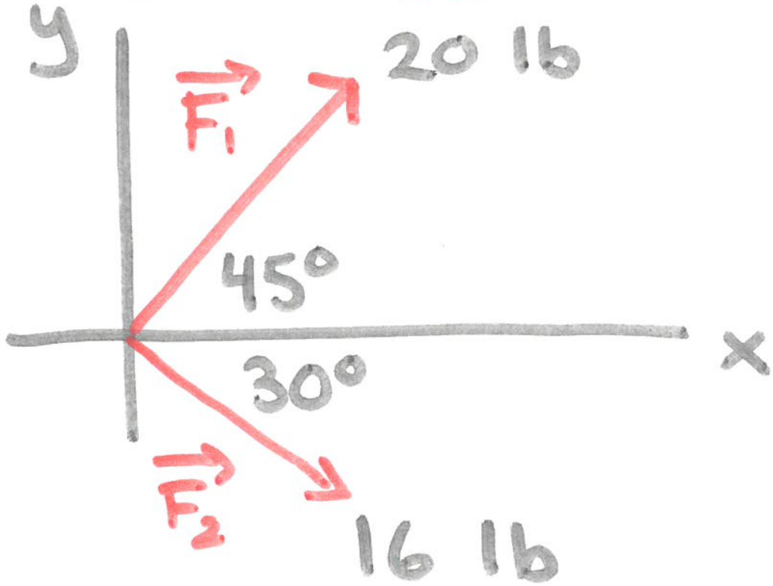
Tension & Resultant Force Problems

Examples from Calculus by James Stewart

Ex 1 Find the magnitude of the resultant force and the angle it makes with the positive x-axis



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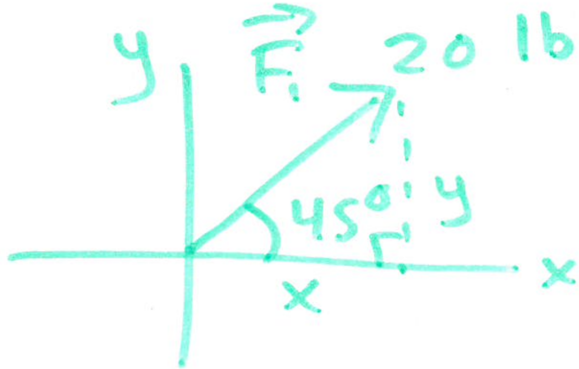
$\vec{F_1}$:



★ Basically we need to break $\vec{F_1}$ up into its horizontal & vertical components

$$\cos 45° = \frac{adj}{hyp} = \frac{x}{\|\vec{F_1}\|}$$

$$x = \|\vec{F_1}\| \cos 45° = 20\left(\frac{\sqrt{2}}{2}\right) = 10\sqrt{2}$$

$$\sin 45° = \frac{opp}{hyp} = \frac{y}{\|\vec{F_1}\|}$$

$$y = \|\vec{F_1}\| \sin 45° = 20\left(\frac{\sqrt{2}}{2}\right) = 10\sqrt{2}$$

$$\vec{F_1} = \langle 10\sqrt{2}, 10\sqrt{2} \rangle$$

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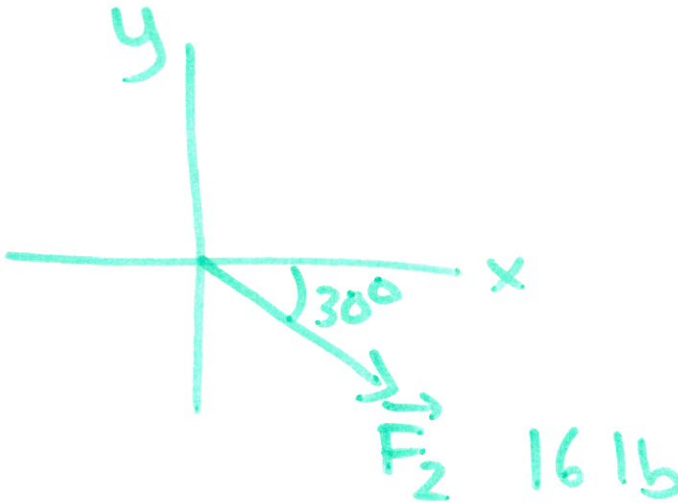

Like the previous problem

$\vec{F_2} = \langle \|\vec{F_2}\| \cos 30°, \|\vec{F_2}\| \sin 30° \rangle$

but with a difference we can see $\vec{F_2}$ is pointing to the right so the x-component should be +

but $\vec{F_2}$ is also pointing down so the y-component should be −

$\vec{F_2} = \langle 16 \cos 30°, -16 \sin 30° \rangle$

$= \langle 16 \sqrt{3}/2, -16 \; 1/2 \rangle = \langle 8\sqrt{3}, -8 \rangle$

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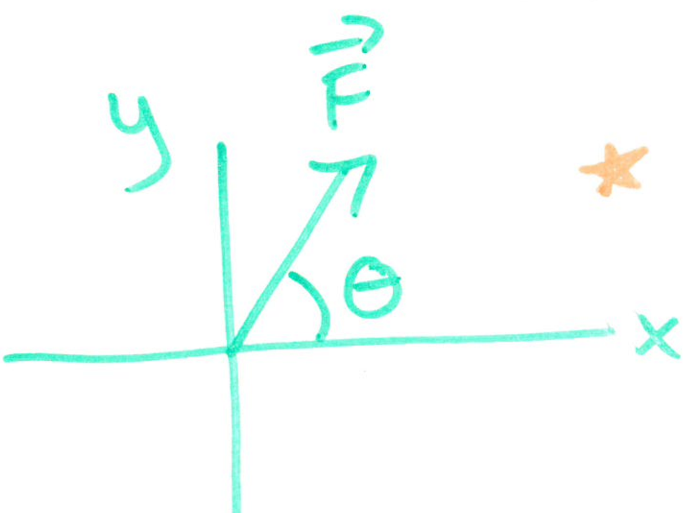
$$\vec{F} = \vec{F_1} + \vec{F_2} = \langle 10\sqrt{2}, 10\sqrt{2} \rangle + \langle 8\sqrt{3}, -8 \rangle$$

$$= \underbrace{\langle 10\sqrt{2} + 8\sqrt{3}, 10\sqrt{2} - 8 \rangle}_{\text{resultant force vector}}$$

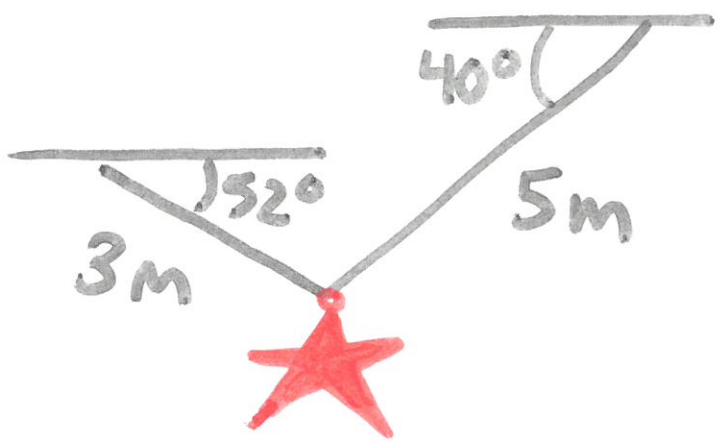
$$\|\vec{F}\| = \boxed{\underbrace{\sqrt{(10\sqrt{2} + 8\sqrt{3})^2 + (10\sqrt{2} - 8)^2}}_{\text{simplify if the numbers are nice}}}$$



* Based on my x & y components of $\vec{F}$, I know it is in the 1st quadrant

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{10\sqrt{2} - 8}{10\sqrt{2} + 8\sqrt{3}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{10\sqrt{2} - 8}{10\sqrt{2} + 8\sqrt{3}}\right)}$$

Ex 2 Ropes 3m & 5m in length are fastened to a star suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of 52° & 40° with the horizontal. Find the tension in each wire & the magnitude of each tension.



Note: The height differences don't matter nor do the lengths of the rope.

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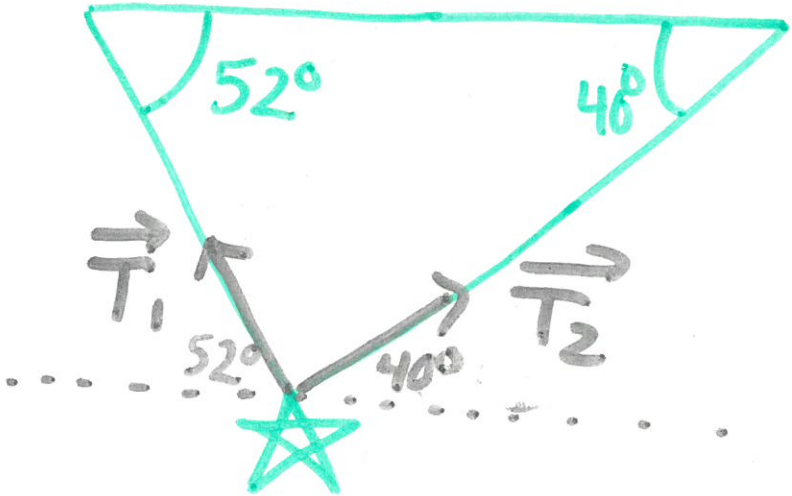


$$\vec{T_1} = \langle -T_1 \cos 52°, T_1 \sin 52° \rangle$$

negative since it is pointing to the left

Let $T_1 = \|\vec{T_1}\|$ & $T_2 = \|\vec{T_2}\|$

$$\vec{T_2} = \langle T_2 \cos 40°, T_2 \sin 40° \rangle$$

$$\vec{T_1} + \vec{T_2} = \langle 0, \underbrace{5(9.8)} \rangle$$

Multiply by gravity since it is in kg to make it a weight.
If it is in lbs or Newtons, don't!

7) The tension is pulling up the same amount that the weight of the star is pulling down.

$$\vec{T_1} + \vec{T_2} = \langle 0, 5(9.8) \rangle$$

$$\langle -T_1 \cos 52^\circ + T_2 \cos 40^\circ, T_1 \sin 52^\circ + T_2 \sin 40^\circ \rangle$$

$$= \langle 0, 5(9.8) \rangle$$

* the x-comp. of the vector on the left equals the x-comp. of the vector on the right.
Likewise with the y-component.

$$-T_1 \cos 52^\circ + T_2 \cos 40^\circ = 0$$

$$T_1 \sin 52^\circ + T_2 \sin 40^\circ = 5(9.8)$$

8] Two equations w/ 2 unknowns

$$T_2 = \frac{T_1 \cos 52^\circ}{\cos 40^\circ}$$

$$T_1 \sin 52^\circ + T_2 \sin 40^\circ = 5(9.8)$$

$$T_1 \sin 52^\circ + \underbrace{T_1 \frac{\cos 52^\circ}{\cos 40^\circ}} \sin 40^\circ = 5(9.8)$$

$$T_1 \left(\sin 52^\circ + \frac{\cos 52^\circ \sin 40^\circ}{\cos 40^\circ} \right) = 5(9.8)$$

$$T_1 \left(\sin 52^\circ + \cos 52^\circ \tan 40^\circ \right) = 5(9.8)$$

$$T_1 = \frac{5(9.8)}{\sin 52^\circ + \cos 52^\circ \tan 40^\circ} \approx 37.56 \text{ N}$$

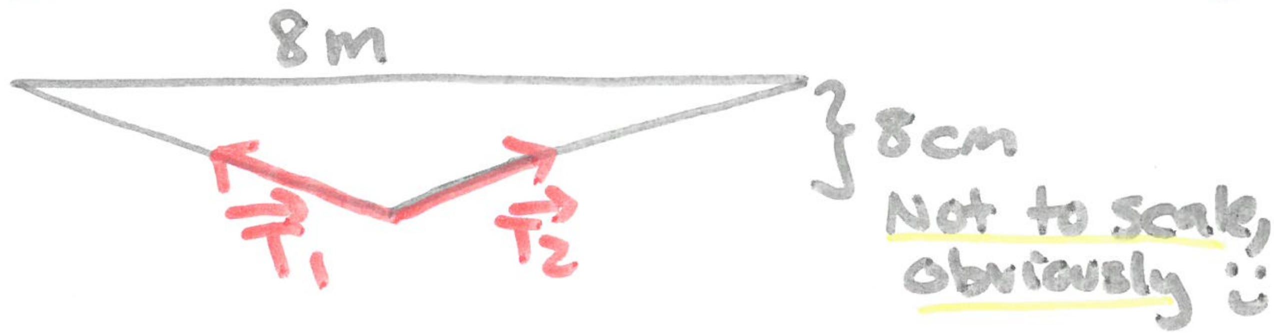
$$T_2 = T_1 \frac{\cos 52^\circ}{\cos 40^\circ} \approx 30.19 \text{ N}$$

9)

$\vec{T_1} = \langle -37.56\cos 52°, -37.56\sin 52° \rangle$

$\vec{T_2} = \langle 30.19\cos 40°, 30.19\sin 40° \rangle$

$\boxed{\text{Ex 3}}$ A clothesline is tied between 2 poles 8m apart. The line is quite taut & has negligible sag. When a wet shirt with a mass of 0.8 kg is hung in the middle of the line, the midpoint is pulled down 8cm. Find the tension in each half of the clothesline.



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☆ Since the shirt is hung in the middle the angle will be the same & $\|\vec{T_1}\| = \|\vec{T_2}\| = T$

$\vec{T_1} = \langle -T\cos\theta, T\sin\theta \rangle$

$\vec{T_2} = \langle T\cos\theta, T\sin\theta \rangle$

$\tan\theta = \frac{opp}{adj} = \frac{.08}{4} = .02$

$\frac{1}{2}(8m)$

$\theta = \tan^{-1}(.02) = 1.146°$

$\vec{T_1} + \vec{T_2} = \langle 0, 0.8\underbrace{(9.8)}_{g} \rangle$

☆ If we hadn't known $\|\vec{T_1}\| = \|\vec{T_2}\|$ we could figure it out here:

$-\underbrace{\|\vec{T_1}\|}_{T}\cos\theta + \underbrace{\|\vec{T_2}\|}_{T}\cos\theta = 0$

$\|\vec{T_1}\|\cancel{\cos\theta} = \|\vec{T_2}\|\cancel{\cos\theta}$ ✓

11 Adding the y-components in the equation $\vec{T_1} + \vec{T_2} = \langle 0, 0.8(9.8) \rangle$

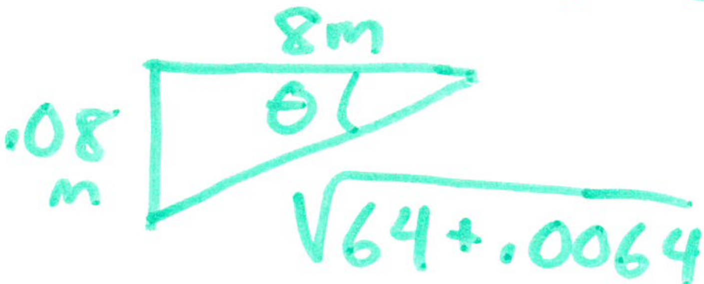
$$T\sin\theta + T\sin\theta = 0.8(9.8)$$

$$2T\sin\theta = .8(9.8)$$

$$T = \frac{.8(9.8)}{2\sin\theta} = \frac{.8(9.8)}{2\sin(1.146°)}$$

magnitude of the tension $\approx 196\text{ N}$

FYI: I didn't have to find θ. I could have done $\sin\theta = \frac{opp}{hyp}$



$$\sin\theta = \frac{opp}{hyp} = \frac{.08}{\sqrt{64.0064}}$$

$\vec{T_1} = \langle -T\cos\theta, T\sin\theta \rangle$

$\approx \langle -195.96, 3.92003 \rangle$

Since the angle θ is very small this y-component is close to $\frac{mg}{2}$, if the angle were bigger this would not be the case!

$\vec{T_2} = \langle T\cos\theta, T\sin\theta \rangle$

$\approx \langle 195.96, 3.92003 \rangle$

$\boxed{\text{Ex 4}}$ The tension $\vec{T}$ at each end of the chain has magnitude 25 N (see the figure). What is the weight of the chain?

13)

$$\vec{T_1} + \vec{T_2} = \langle 0, \underbrace{\text{weight}}_{\text{if they asked for mass of the chain we would put } mg \text{ here}} \rangle$$

$\|\vec{T_1}\| = \|\vec{T_2}\| = T$
since angles are the same

$$\vec{T_1} = \langle -T\cos 37^\circ, T\sin 37^\circ \rangle$$
$$\vec{T_2} = \langle T\cos 37^\circ, T\sin 37^\circ \rangle$$
$$\vec{T_1} + \vec{T_2} = \langle 0, \text{weight} \rangle$$

x-components cancel

y-components: $T\sin 37^\circ + T\sin 37^\circ = \text{weight}$

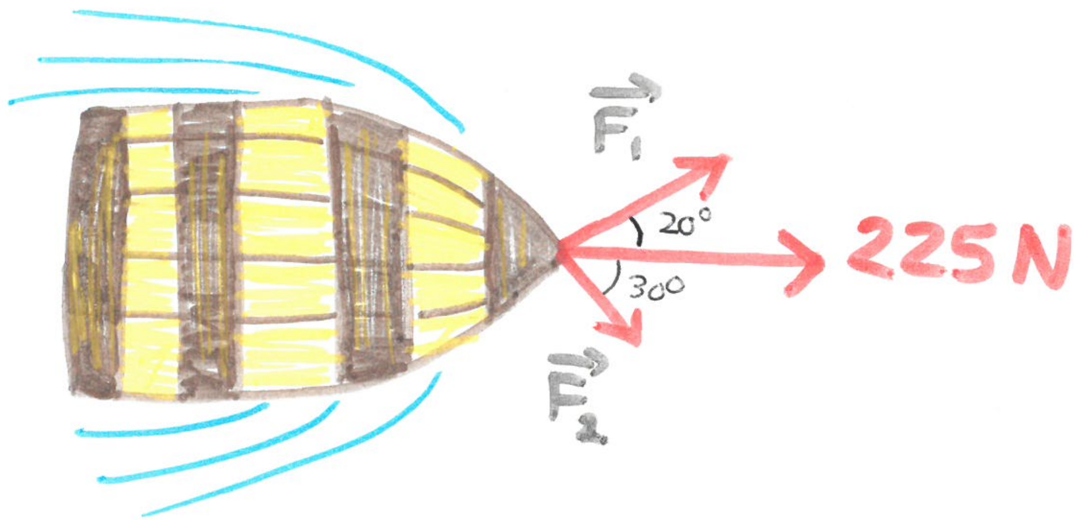
$$2T\sin 37^\circ = \text{weight}$$

$$\boxed{\text{Weight} = 2\underbrace{(25)}_{\text{I forgot they gave us } \|\vec{T}\| \text{ until now}}\sin 37^\circ \text{ N}}$$

FYI!

$$\text{Mass} = \frac{2(25)\sin 37^\circ}{9.8} \text{ kg}$$

Ex 5 A boat is pulled onto shore using two ropes, as shown in the diagram. If a force of 255 N is needed, find the magnitude of the force in each rope.



The resultant force has a magnitude of 225 N. Since it is all going to the right, resultant force = $\langle 225, 0 \rangle$

You can also figure this since $\theta = 0$.

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$$\vec{F_1} + \vec{F_2} = \langle 225, 0 \rangle$$

$$\vec{F_1} = \langle \underbrace{F_1}\cos 20°, \underbrace{F_1}\sin 20° \rangle$$

$$\|\vec{F_1}\| = F_1$$

$$\vec{F_2} = \langle F_2 \cos 30°, -F_2 \sin 30° \rangle$$

$$\|\vec{F_2}\| = F_2$$

y-component negative since arrow pointing down. Could also figure this out if we used $\theta = -30°$

$$\vec{F_2} = \langle F_2 \sqrt{3}/2, -F_2 \tfrac{1}{2} \rangle$$

$$\vec{F_1} + \vec{F_2} = \langle 225, 0 \rangle$$

$$\langle F_1 \cos 20° + F_2 \sqrt{3}/2, F_1 \sin 20° - F_2/2 \rangle = \langle 225, 0 \rangle$$

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$$F_1 \cos 20^\circ + F_2 \frac{\sqrt{3}}{2} = 225$$

$$F_1 \sin 20^\circ - F_2/2 = 0$$

$$F_1 = \frac{F_2}{2 \sin 20^\circ}$$

$$F_1 \cos 20^\circ + F_2 \frac{\sqrt{3}}{2} = 225$$

$$\frac{F_2}{2 \sin 20^\circ} \cos 20^\circ + F_2 \frac{\sqrt{3}}{2} = 225$$

$$F_2 = \frac{225}{\frac{\cos 20^\circ}{2 \sin 20^\circ} + \sqrt{3}/2} \approx 100.46\ N$$

$$F_1 = \frac{F_2}{2 \sin 20^\circ} = \frac{\frac{225}{\frac{\cos 20^\circ}{2 \sin 20^\circ} + \frac{\sqrt{3}}{2}}}{2 \sin 20^\circ} \approx 146.86\ N$$