MA 241 - 050 Test 1 Version 2 Show all of your work

Use of the internet beyond our moodle site beyond accessing our class notes is cheating and will carry all the penalties of cheating

1. (12 points) Suppose that $f(1) = 3$, $f(4) = 5$, $f'(1) = 6$, $f'(4) = 4$, and f'' is continuous.

Find the value of $\int_1^4 x\, f''(x)dx$

2. (12 points) Find the length of the curve given by $y = \frac{x^2}{4} - \frac{1}{2}\ln(x)$ where $1 \le x \le 2$

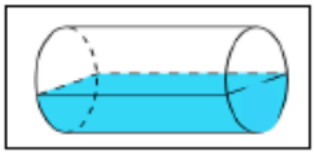
3. (12 points) A spring has a natural length of 10 inches. A 4 lb force stretches the spring to 13 in. Find the work done in stretching the spring from 12 in. to 16 in. Include units with your answer.

4. (12 points) Find the average amount of liquid in a 4000 gallon tank while it is draining if it is known that after t minutes the amount of liquid in the tank is $V(t) = 40(10 - t)^2$

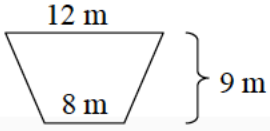
5. (12 points) A 5 meter chain with a weight of 30 N is lying on the ground. Find the work needed to raise one end to a height of 4 meters. Your answer should include units and a picture indicating the locations of your x - axis and y - axis.

6. (15 points) Find the centroid of the triangular lamina with vertices (0,0), (3,2), and (3,0)

7. (12 points) A horizontal cylindrical tank is half full of water. Given the weight density of water is 62.4 lb/ft^3. Set up (**Do not evaluate**) the integral needed to find the work to pump all of the water out of the top of the tank. The tank is 7 ft long and the sides have a 6 ft diameter as shown below. Your answer should include units and a picture indicating the locations of your x - axis and y - axis.



8. (13 points) A tank is being filled with water. The tank is sturdy except for trapezoidal windows located at either end of the tank (one of these windows is pictured below). These windows can only withstand a maximum force of 80 N. Set up (**Do not evaluate**) the integral and equation needed to determine how high the water can reach before the trapezoidal sides shatter. Your answer should include units and a picture indicating the locations of your x - axis and y - axis. You may wish to use $1000 kg/m^3$ as the density of water and $g = 9.8 m/s^2$



C2 T1 Solutions

1. (12 points)

$\int_1^4 x f''(x)\,dx$ LIATE

$u = x \quad v = f'(x)$

$du = dx \quad dV = f''(x)\,dx$

$= uv - \int v\,du$

$= x f'(x)\Big|_1^4 - \int_1^4 f'(x)\,dx$

$= 4f'(4) - 1f'(1) - f(x)\Big|_1^4$

$4(4) - 1(6) - f(4) + f(1)$

$10 - 5 + 3 = \boxed{8}$

2. (12 pts)

$\frac{dy}{dx} = \frac{1}{2}x - \frac{1}{2}\frac{1}{x}$

$L = \int_1^2 \sqrt{\left(\frac{1}{2}x - \frac{1}{2}\frac{1}{x}\right)^2 + 1}\,dx$

$= \int_1^2 \sqrt{\frac{1}{4}x^2 - 2\left(\frac{1}{2}x\right)\left(\frac{1}{2}\frac{1}{x}\right) + \frac{1}{4}\frac{1}{x^2} + 1}\,dx$

$= \int_1^2 \sqrt{\left(\frac{1}{2}x + \frac{1}{2}\frac{1}{x}\right)^2}\,dx$

$= \int_1^2 \frac{1}{2}x + \frac{1}{2}\frac{1}{x}\,dx = \frac{1}{4}x^2 + \frac{1}{2}\ln x\Big|_1^2$

$= 1 + \frac{1}{2}\ln 2 - \frac{1}{4} - \frac{1}{2}\ln 1$

3. (12 pts)

$F = kx$

$4 = k\left(\frac{1}{4}\right) \quad k = 16$

($\frac{1}{4}$ ↖ 3 inches)

$$W = \int_{\frac{1}{6}}^{1/2} 16x\,dx = 8x^2\Big|_{1/6}^{1/2} = 8\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{6}\right)^2\right] \text{ ft-lb}$$

4. (12 pts)

$v(t) = 40(10-t)^2 = 0 \rightarrow t = 10$

$$\frac{1}{10-0}\int_0^{10} 40(10-t)^2\,dt \qquad u = 10-t, \quad du = -dt$$

$$-\frac{1}{10}\int_{10}^{0} 40u^2\,du = \frac{1}{10}\int_0^{10} 40u^2\,du$$

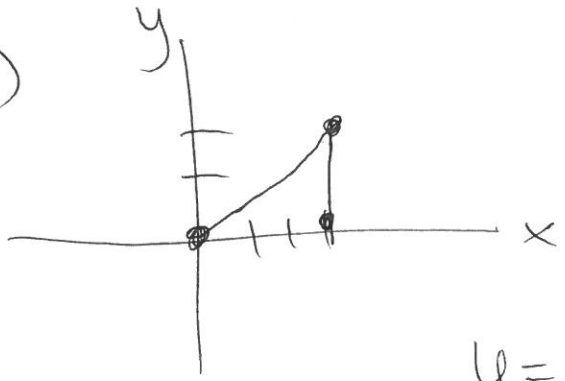
$$\frac{4}{3}u^3\Big|_0^{10} = \boxed{\frac{4}{3}\cdot 10^3}$$

5. (12 pts)

$$W_{piece} \approx F d = \left(\frac{30N}{5m}\Delta y\right)(y)$$

$$W = \int_0^4 6y\,dy = 3y^2\Big|_0^4 = \boxed{3(16) = 48\ J}$$

6. (15 pts)



$$m = \frac{\Delta y}{\Delta x} = \frac{2-0}{3-0} = \frac{2}{3}$$

$$y = \frac{2}{3}x$$

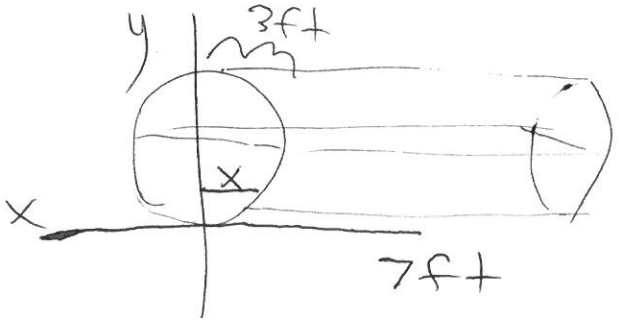
$$\bar{x} = \frac{\int_0^3 x\left(\frac{2}{3}x\right)dx}{\int_0^3 \frac{2}{3}x\,dx} = \frac{\frac{2}{9}x^3\Big|_0^3}{\frac{1}{3}x^2\Big|_0^3} = \frac{6}{3} = 2$$

$$\bar{y} = \frac{\int_0^3 \frac{1}{2}\left(\frac{2}{3}x\right)^2 dx}{3} = \frac{\int_0^3 \frac{4}{2(9)}x^2\,dx}{3}$$

$$= \frac{\frac{2}{27}x^3\Big|_0^3}{3} = \frac{2}{3}$$

$$(\bar{x}, \bar{y}) = \boxed{\left(2, \frac{2}{3}\right)}$$

7. (12 pts)



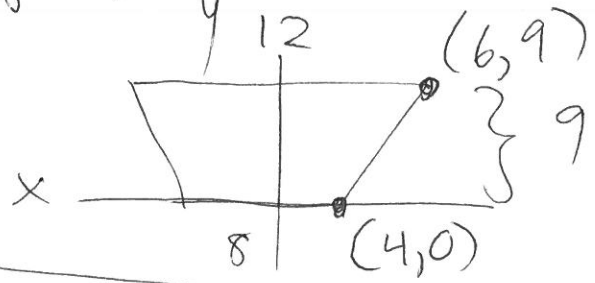
$(x-0)^2+(y-3)^2=9$

$x=\sqrt{9-(y-3)^2}$

$W=\int_a^b \rho A(y)(h-y)\,dy$

$=\int_0^3 62.4\underbrace{\left[2\sqrt{9-(y-3)^2}\right](7)}_{A(y)}(6-y)\,dy$ ft-lbs

8. (13 pts)



$m=\frac{9-0}{6-4}=\frac{9}{2}$

$y=\frac{9}{2}x+b$

$(4,0)$

$0=-18+b \rightarrow b=-18$

$80=\int_0^b 1000(9.8)\left[2\left(\frac{2}{9}(y+18)\right)\right](b-y)\,dy$

$y=\frac{9}{2}x-18$

$x=\frac{2}{9}(y+18)$