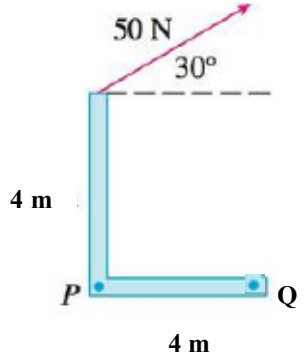
Test 1 Version 1

1. (25 points) Use the points P(1,2,3), Q(1,3,6), R(3,8,6), and S(3,7,3) to answer the following:
   a) Find the vector representation of the line segment from P to Q
   b) Find the area of the parallelogram with vertices PQRS
   c) Find the distance from point P to R

2. (29 points) Use the lines $\mathbf{r}_1(t) = \langle 12 - 4t,\ 15,\ t + 3 \rangle$ and $\mathbf{r}_2(s) = \langle 5 - s,\ 3s,\ 2s - 4 \rangle$ to answer the following:
   a) Find the point where the lines intersect
   b) Find the plane that contains these lines
   c) Find the angle between the lines
   d) Find two vectors with a magnitude of 3 that are parallel to the direction vector of line $\mathbf{r}_2(s)$

3. (15 points) A rock bounces off the surface of an unknown planet with an angle of elevation of 45° and an initial speed of 20ft/s. Use the techniques we've discussed in class, to find the following:
   a) Find the velocity vector **v** (use g to represent the gravity on this planet).
   b) Use your work from part a) to find the position vector **r**. You can have g in your answer.
   c) Find the gravity g on that planet if the rock lands 100ft away from its starting point.

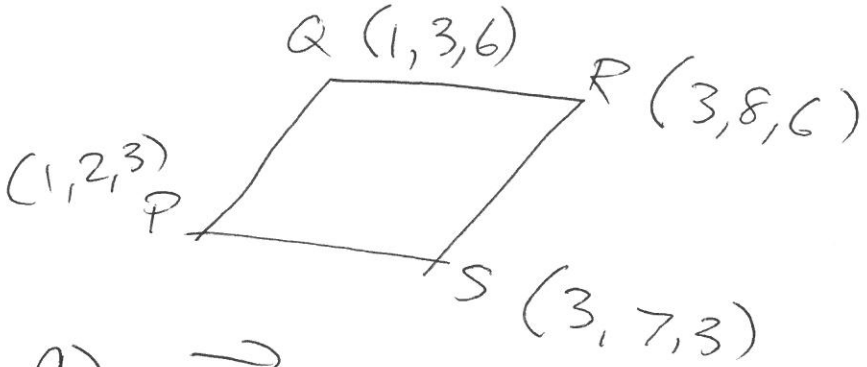
4. (15 points) Find the magnitude of the torque about point P and then determine the magnitude of the torque about point Q



5. (16 points) The tension at each end of the chain has a magnitude 24 N. Find the tension vector $\vec{\mathbf{T}}_1$ on the left side of the chain and then find the mass of the chain. You may wish to use $g = 9.8 m/s^2$

C3 T1 Solutions

1. (25 points)



a) $\overrightarrow{PQ} = \langle 0, 1, 3 \rangle$

$$\boxed{\vec{r} = \langle 1,2,3 \rangle + \langle 0,1,3 \rangle t, \quad 0 \le t \le 1}$$

b) $\overrightarrow{PS} = \langle 2, 5, 0 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 3 \\ 2 & 5 & 0 \end{vmatrix} = \langle 0-15, -(0-6), 0-2 \rangle$$

$$= \langle -15, 6, -2 \rangle$$

$$\|\overrightarrow{PQ} \times \overrightarrow{PS}\| = \boxed{\sqrt{15^2 + 6^2 + 2^2}}$$

c)

$$\sqrt{(3-1)^2 + (8-2)^2 + (6-3)^2}$$

$$= \sqrt{4 + 36 + 9} = \sqrt{49} = \boxed{7}$$

2. (29 pts)

a) $12-4t=5-s$

$15=3s \rightarrow s=5$

$t+3=2s-4$ $\quad 6=t+3 \quad t=3$

pt: $(0,15,6)$

b) $\vec{n}=\vec{v}_1\times\vec{v}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & 1 \\ -1 & 3 & 2 \end{vmatrix}$

$=\langle 0-3, -(-8+1), -12\rangle$

$=\langle -3, 7, -12\rangle$

$$\boxed{-3(x-0)+7(y-15)-12(z-6)=0}$$

c) $\vec{a}\cdot\vec{b}=\|\vec{a}\|\,\|\vec{b}\|\cos\theta$

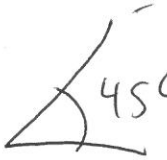
$\langle -4,0,1\rangle\cdot\langle -1,3,2\rangle=\sqrt{17}\sqrt{1+9+4}\cos\theta$

$4+0+2$

$$\boxed{\theta=\cos^{-1}\left(\frac{6}{\sqrt{17}\sqrt{14}}\right)}$$

d) $$\boxed{\pm 3\,\frac{\langle -1,3,2\rangle}{\sqrt{14}}}$$

5. (15 pts)



a) $\vec{a} = \langle 0, 0, -g \rangle$

$\vec{v} = \langle 0, 0, -gt \rangle + \vec{c}$

$\vec{v}(0) = \langle 20\cos 45°, 0, 20\sin 45° \rangle$

$= \langle 10\sqrt{2}, 0, 10\sqrt{2} \rangle$

$\vec{v} = \langle 10\sqrt{2}, 0, 10\sqrt{2} - gt \rangle$

b) $\vec{r} = \langle 10\sqrt{2}t, 0, 10\sqrt{2}t - \frac{1}{2}gt^2 \rangle + \vec{0}$

c)

$10\sqrt{2}t = 100$

$10\sqrt{2}t - \frac{1}{2}gt^2 = 0$

$\left(10\sqrt{2} - \frac{1}{2}gt\right)t = 0$

↖ start of ground

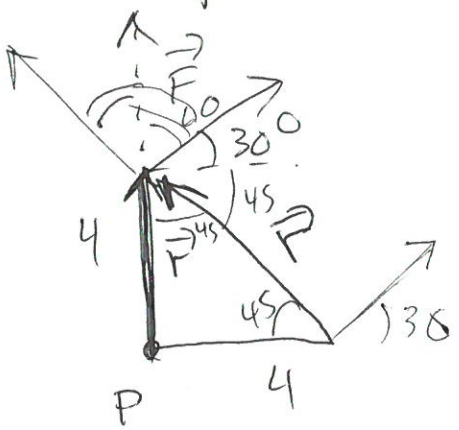
$10\sqrt{2} - \frac{1}{2}g\left(\frac{100}{10\sqrt{2}}\right) = 0$

$10\sqrt{2} = \frac{10}{2\sqrt{2}}g$

$\frac{10\sqrt{2} \cdot 2\sqrt{2}}{10} = g$

$\boxed{g = 4 \text{ ft/s}^2}$

4. (15 pts)



P: $\|\vec{\tau}\| = \|\vec{r}\| \, \|\vec{F}\| \sin\theta$

$= 4(50)\sin 60^\circ$

$= \boxed{200\sqrt{3}/2}$

Q: $\|\vec{\tau}\| = \|\vec{r}\| \, \|\vec{F}\| \sin\theta$

$= \boxed{\sqrt{32}\ 50 \sin(180-75^\circ)}$

5. (16 pts)

$\vec{T}_1 = \langle -24\cos 60^\circ, 24\sin 60^\circ \rangle$

$\boxed{\vec{T}_1 = \langle -12, 12\sqrt{3} \rangle}$

$\vec{T}_1 + \vec{T}_2 = \langle 0, mg \rangle$

$24\sqrt{3} = mg$

$\boxed{m = \frac{24\sqrt{3}}{9.8}}$